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ACPS

Alexandria City Public Schools

Mathematics Framework: Research-Based Strategies to Promote High Levels of Student Achievement

Mathematics Framework

Alexandria City Public Schools

Statement of Purpose:

To support mathematical proficiency and the acquisition of 21st century skills, Alexandria City Public Schools (ACPS) has developed the following mathematics framework. It aligns with the Literacy Framework by providing a synthesis of research-based principles and strategies proven effective in promoting all students' mathematics development—including the critical, creative, and self-regulated thinking processes that underlie true 21st Century literacy and mathematics.

21st century mathematics calls for a shift to focus on sense-making, reasoning, and connections to real-world situations. Students will need knowledge and skills that prepare them to apply mathematics in a variety of contexts, including their future lives as responsible citizens. A transformation is required that results in a greater emphasis on the many ways that math helps us understand the world, and less on math for its own sake. There needs to be a focus on understanding and concepts, not just computation or procedures.

Looking ahead to the 21st century, developing and applying real-world situations will require new technology tools and new approaches to teaching and learning. It will also require new assessment methods. The goal of the assessments should be to inform students and teachers about the level of understanding achieved, and of the next necessary steps in instruction. Ongoing informal assessment that guides teaching and learning brings about increased learning as well as increased self-esteem for students.

Students will need the resources to prepare them for our rapidly changing world. By working on authentic tasks and real-life problem situations, students make connections related to their own learning of mathematics as well as important new connections among graphic, symbolic, and dynamic representations that are critical in order to understand mathematics effectively. They will also need to recognize that studying mathematics in high school is important for their future careers.

A commitment to teacher professional development is essential that is collaborative with time allotted for vertical discussions and alignment across grade levels and high school courses. Teachers will need long-term professional development and support, including opportunities for reflection on their practice and guidance in improving it.

To achieve the vision of reasoning and sense-making as the focus of students' mathematical experiences, all components of the educational system – curriculum, instruction, and assessment – must work together and be designed to support students' achieving these concepts and skills. Through a coherent and cohesive mathematics program with a strong alignment of curriculum, instruction and assessment, students will have the opportunity to be fully prepared for the challenges of the 21st century workplace.

Long-Range Goals of This Framework

This Alexandria City Public Schools mathematics framework is designed to:

1. Articulate an operational definition for 21st Century mathematics.

2. Delineate a set of principles related to the key elements of mathematics (i.e., "trans-disciplinary" literacy and mathematics principles and strategies).

3. Summarize key research conclusions and evidence-based best practices related to mathematics.

4. Provide an overview of observable subject-specific strategies for use in different content areas.

5. Delineate strategies designed for use with students as they move along a learning continuum (from emergent through advanced, independent applications, and transfer of mathematics principles and techniques).

6. Provide a rich variety of electronic/downloadable resources for use with mathematics professional development sessions and workshops (e.g., websites, electronic links, templates, assessment inventories and other tools).

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A Vision for School Mathematics
(National Councils of Teachers of Mathematics, 2000)

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.

Why This Mathematics Framework?
What Is It Designed to Help Teachers Do?

Suggestions for Using This Mathematics Framework:

1. This mathematics framework should be used in the context of an ongoing professional learning community (PLC) discussion of how to improve students' mathematics performance within and across the content areas.

2. A professional learning community should consist of small groups of individuals committed to working on a variation of study group and action research processes. Key guiding questions for mathematics PLCs should include:

    - What is the status of our current students' mathematics performance?

    - To what extent are there gaps or areas of underachievement related to mathematics?

    - When we disaggregate student performance data, do we discover specific cohorts of students who need extra support or intervention in the areas of mathematics?

    - What research-based strategies for promoting mathematics are we willing to try out to address our identified problems and areas of achievement gap?

    - How can we study, collect data, and determine the "value added" of the strategies and interventions we use?

    - How can we share our conclusions and insights with others in the school?

3. When PLC groups identify emerging issues and problems shared with other educators in the school, the mathematics framework can be used as a rich resource for coaching and instructional rounds. Here are some suggestions for both:

    - <u>Instructional Coaching</u>: Instructional coaches are currently available at kindergarten through grade 8 to support teachers in implementing the practices presented in this Mathematics Framework. They can do informal classroom observations, data analysis, and provide demonstration lessons modeling key literacy strategies. Peers can also become fellow coaches, observing one another's classrooms and then exploring ways in which strategies from the Mathematics Framework might enhance student performance in the observed classrooms.

    - <u>Instructional Rounds</u>: Small groups make visitations to classrooms, with agreed-upon "look-fors" (both in terms of what the teacher is doing and the students are doing…) derived from the Mathematics Framework. They are responsible for collecting and analyzing data patterns relative to the criteria observed.

Instructional rounds are designed for data analysis and drawing of inferences about ways to improve student achievement, rather than for purposes of evaluation.

4. Additionally, this Mathematics Framework can be used for a variety of formal and informal professional development purposes, including:

- Study group discussions
- Strategic planning activities (including coaching sessions focusing on mathematics)
- Departmental or grade level professional development
- Formal professional development (on the entire mathematics framework or key aspects of it such as “writing to learn” strategies)
- Administrative observation trainings
- Professional development activities related to Response to Intervention (RtI)
- Alignment with school improvement planning initiatives
- Outreach initiatives to parents and community members

Observing Mathematics at Work in ACPS Classrooms: A Comprehensive Observation Protocol

Suggestions for Use:

The following observation protocol contains a detailed description of what an observer should see in any ACPS classroom in which mathematics-based principles and best practices are operational. This protocol can be used for a variety of purposes, including:

1. Informal observations by administrators
2. Peer observations and coaching
3. Instructional rounds
4. Walk-through observations involving external and/or internal teams

The observation categories and related performance indicators can be addressed in their totality—or observers may elect to concentrate upon one category at a time, looking for patterns and trends related to a single area of mathematics instruction and related student performance.

Observers may wish to use the following rating scale to assess the level of use of a particular strategy or category:

4=Highly evident throughout the lesson
3=Evident during key aspects of lesson delivery
2=Occasionally evident
1=Little if any evidence

Part I: Mathematics Competencies—General Strategies: *To what extent do teachers:*

Consistently Use the Following General Mathematics Strategies:

1. Activate students' prior knowledge, explaining mathematical concepts and facts in terms of simpler concepts and facts tied to students' prior learning.

2. Use multiple representations (i.e., two- and three-dimensional visual models and manipulatives) to help students make abstractions more concrete and comprehensible.

3. Model and encourage students to try out alternative approaches to mathematical problem solving, including regular use of estimation strategies.

4. Use ongoing assessments to provide on-the-spot feedback to students, encouraging them to monitor their own progress toward achieving desired results.

5. Teach for transfer, helping students to make the connections they otherwise might not make, and helping them to cultivate mental habits of connection-making.

6. Encourage the students to go beyond the information given, tackling some task of justification, explanation, example-finding or the like that reaches further than anything in the textbook or the lecture.

7. Encourage student to think like a mathematician - justify, explain, solve problems, and manage inquiry within the discipline.

Part II: Instructional "Shifts" That Promote Mathematics Achievement for All Learners (Identified by Steve Leinwand in his book *Accessible Mathematics* [2009]):

1. Determine clear instructional and learning priorities in mathematics:

   - Core mathematics concepts
   - Key skills with understanding
   - Facility with terms, vocabulary, and notation
   - Ability to apply the mathematics and solve problems

2. Emphasize key (and recurrent) mathematics instructional priorities:

   - Ask "Why?" throughout a mathematics lesson, engaging students in "accountable talk" and metacognitive reflection and self-assessment.
   - Provide alternative approaches to learning (based upon students' varying readiness levels, interests, and learning profiles).
   - Present mathematical concepts and procedures via multiple representations and formats.
   - Reinforce language-rich classrooms (with key mathematics concepts and big ideas emphasize via classroom discourse, interactive word walls, etc.).
   - Emphasize contexts (i.e., how mathematics is used to solve real-world problems within the classroom—and beyond it).
   - Make connections for students (i.e., Why are we learning this? Why should we care about it? How is it used in the "real-world"?)
   - Use advance organizers to preview new content.

3. Demonstrate ongoing sensitivity to a core essential question: Why should we bother?

   - Support sense-making for all (i.e., moving beyond the acquiring phase of learning into constructed meaning, guided transfer, and independent transfer.
   - Support understanding by all students (focusing on students' growing ability to explain, apply, and interpret using mathematical concepts, algorithms, procedures, and reasoning processes).

- Mathematically empower students, helping all learners to develop a sense of mathematical efficacy.

4. Use ongoing review (including periodic cumulative reviews) to help students retain key concepts and big ideas.

   - Conduct ongoing cumulative reviews, ensuring that students understand what they are required to retain—and apply it with growing levels of constructed meaning and both guided and independent transfer.
   - Use cumulative review to keep skills and understandings fresh, reinforce previously taught material, and give students a chance to clarify their understandings.
   - Use a brief review and whole-class checking of "mini-math" questions as an opportunity to re-teach when necessary.

5. Adapt what we know works in our reading programs and apply it to mathematics instruction:

   - Consistent parallels are apparent between the types of questions that require inferential and evaluative comprehension in reading instruction and the probing for ways in which the answers were found, alternative approaches, and reasonableness in mathematics instruction.
   - All numerical and one-word answers are consistently greeted with a request for justification.
   - Only reasonable homework assignments are given, and when homework is reviewed, the focus is on explanation and understanding, not on checking for right answers.

6. Use multiple representations of mathematical entities:

   - Frequent use of pictorial representations to help students visualize the mathematics they are learning.
   - Frequent use of the number line and bar models to represent numbers and word problems.
   - Frequent opportunities for students to draw or show and then describe what is drawn or shown.

7. Create language-rich mathematics classrooms:

   - An ongoing emphasis on the use and meaning of mathematical terms, including their definitions and their connections to real-world entities and/or pictures
   - Student and teacher explanations that make frequent and precise use of mathematics terms, vocabulary, and notation

- An extensive use of word walls that capture the key terms and vocabulary with pictures when appropriate and in English as well as Spanish when appropriate

8. Take every available opportunity to support the development of number sense:

    - Sustain an unrelenting focus on estimation and justifying estimates to computations and to the solution of problems.
    - Promote an unrelenting focus on a mature sense of place value.
    - Encourage frequent discussion and modeling about how to use number sense to “outsmart” the problem.
    - Provide students with frequent opportunities to put the calculator aside and estimate or compute mentally when appropriate.

9. “Milk the Data”: Build from graphs, charts, and tables:

    - Regularly use problems drawn from the data presented in tables, charts, and graphs.
    - Provide regular opportunities for students to make conjectures and draw conclusions from data presented in tables, charts, and graphs.
    - Present problems requiring frequent conversion, with and without technology, of data in tables and charts into various types of graphs, with discussions of their advantages, disadvantages, and appropriateness.

10. Increase the natural use of measurement throughout the curriculum, tying learning to such questions as: How big? How much? How far?

    - Use lots of questions are included that ask: How big? How far? How much? How many?
    - Make measurement an ongoing part of daily instruction and the entry point for a much larger chunk of the curriculum.
    - Frequently ask students to find and estimate measures, to use measuring, and to describe the relative size of measures that arise during instruction.
    - Offer frequent reminders that much measurement is referential – that is, we use a referent (such as your height or a sheet of paper) to estimate measures.

11. Minimize what is no longer important, and teach what is important when it is appropriate to do so:

    - Provide a mathematics curriculum comprised of skills, concepts, and applications that are reasonable to expect all students to master—and <u>not</u> those skills, concepts, and applications that have gradually been moved to an earlier grade on the basis of inappropriately raising standards.
    - Implement a district curriculum that includes essential skills and understandings for a world of calculators and computers—and not what many recognize as too much content to cover at each grade level

- Maintain deliberate questioning of the appropriateness of the mathematical content, regardless of what may or may not be on the high-stakes state test, in every grade and course.

12. Embed the mathematics in realistic problems and real-world contexts:

- Frequently embed the mathematical skills and concepts in real-world situations and contexts.
- Frequently use questions asking: “So what questions arise from these data or this situation?”

Part III: Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics

Randy Charles (National Councils for School Mathematics, 2005)

The Big Ideas of Number and Number Sense:

Numbers: The set of real numbers is infinite, and each real number can be associated with a unique point on a number line.

Counting Numbers:

- Counting tells how many items there are altogether. When counting, the last number tells the total number of items; it is a cumulative count.
- Counting a set in a different order does not change the total.
- There is a number word and a matching symbol that tell exactly how many items are in a group.
- Each counting number can be associated with a unique point on the number line, but there are many points on the number line that cannot be named by the counting numbers.

The Base-Ten Numeration System: The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value. Examples of Mathematical Understandings:

- **Whole Numbers:**

  1. Numbers can be represented using objects, words, and symbols.
  2. For any number, the place of a digit tells how many ones, tens, hundreds, and so forth are represented by that digit.
  3. Each place value to the left of another is ten times greater than the one to the right (e.g., 100 = 10 x 10).

- **Decimals:**

  1. Decimal place value is an extension of whole number place value.
  2. The base-ten numeration system extends infinitely to very large and very small numbers (e.g., millions & millionths).

Comparison: Numbers, expressions and measures can be compared by their relative values.

Equivalence: Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

The Big Ideas of Computation and Estimation:

Operational Meaning and Relationships: The same number sentence can be associated with different concrete or real-world situations, AND different number sentences can be associated with the same concrete or real-world situations.

Basic Facts and Algorithms: Basic facts and algorithms for operations with rational numbers use notions of equivalence to transform calculations into simpler ones.

Estimation: Numerical calculations can be approximated by replacing numbers with other numbers that are close and easy to compute with mentally. Measurements can be approximated using known referents as the unit in the measurement process.

The Big Ideas of Geometry:

Shapes and Solids: Two- and three-dimensional objects with or without curved surfaces can be described, classified, and analyzed by their attributes.

- Point, line, line segment, and plane are the core attributes of space objects, and real-world situations can be used to think about these attributes.
- There is more than one way to classify most shapes and solids.

The Cartesian Coordinate System: Every point in the plane can be described uniquely by an ordered pair of numbers; the first number tells the distance to the left or right of zero on the horizontal number line; the second tells the distance above or below zero on the vertical number line.

Congruent Figures: Congruent figures remain congruent through translations, rotations, and reflections.

The Big Ideas of Measurement:

Estimation: Numerical calculations can be approximated by replacing numbers with other numbers that is close and easy to compute with mentally. Measurements can be approximated using known referents as the unit in the measurement process.

Measurement: Some attributes of objects are measurable and can be quantified using unit amounts.

Proportionality: If two quantities vary proportionally, that relationship can be represented as a linear function.

The Big Ideas of Probability and Statistics:

Data Collection: Some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it.

Data Representation: Data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation.

Data Distribution: There are special numerical measures that describe the center and spread of numerical data sets.

The Big Ideas of Patterns, Functions, and Algebra:

Equivalence: Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

Properties: For a given set of numbers there are relationships that are always true, and these are the rules that govern arithmetic and algebra.

Patterns: Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.

Variable: Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.

Proportionality: If two quantities vary proportionally, that relationship can be represented as a linear function.

Relations and Functions: Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.

Equations and Inequalities: Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

<u>Part IV</u>: Meta-Strategies (Organized by Strands in Mathematics):

The following are strategies teachers can use to help guide students to a significant understanding of key concepts (i.e., “big ideas) taught over the course of multiple grade levels and disciplines with mathematics. Each set of strategies is associated with the overarching themes of mathematics instruction in Virginia, which are woven throughout the K – 12 curriculum:

1. **Numbers and Number Sense**
2. **Computation and Estimation**
3. **Geometry**
4. **Measurement**
5. **Probability and Statistics**
6. **Patterns, Functions and Algebra**

Each of the strands and their subcategories are associated with Randy Charles’s “Big Ideas” of mathematics education (NCSM Journal, spring, 2005), and each strategy is associated with a meta-strategy described in the following section. The strategies follow a continuum of learning where students work between varying levels of understanding to fully understand a concept. The levels on this continuum are: (1) acquisition of key concepts and skills; (2) constructing meaning and making connections to new learning; and (3) transferring understanding to novel tasks.

Ultimately, the goal of these strategies is to serve as a resource for teachers looking for ideas to improve or enhance their method of teaching. Further, it will serve as an instructional tool for teachers to better learn what it means for students to acquire key concepts, construct meaning, and transfer their understanding of a mathematical topic to a novel situation.

One way a teacher can use this resource is to view it like she would an encyclopedia. Typically, a person would not read an encyclopedia from cover-to-cover but look up a specific subject in which she is interested. Likewise, a teacher can use this resource to look up a specific strand to which she is teaching and go directly to it for ideas.

Categories of Meta-Strategies for Mathematics

- **<u>Manipulatives</u>:** Two- and three-dimensional models and representations, encouraging students to synthesize and make connections (e.g., a terminology collage, a big concept mobile)

- **<u>Non-Linguistic Representations</u>:** Concept maps, idea webs, dramatizations, and other forms of visual representations of key concepts, patterns, and big ideas.

- **Investigation and Inquiry-Based Learning:** Students either discover for themselves the results of a learning goal, or the teacher guides them to the desired learning goal but without making it explicit what it is.

- **Success Criteria for Self- and Peer-Evaluation:** Students discuss and critique their own work or the work of others.

- **Accountable Talk:** Creating social norms for `talk' that model cognitive norms of smart people; that is, to translate mental habits of smart people into social habits for all students. These habits of talk get internalized and all the students start talking and thinking like smart students.

- **Multiple Representations:** Students generate several ways of representing a problem. For example, solutions to equations can be found algebraically, in tables, and on graphs.

- **Comparing and Contrasting:** Students compare methods of solving problems with one another and discuss the positive and negative differences of each method.

- **Connections Between Concrete and Symbolic:** Students learn that mathematics is the language of structure by working through and within tangible models before tying algebraic representations to them. For example, Algebra I students can determine where a phone plan charging a $5 fee and $0.10 a minute costs the same as plan charging an $8 fee and $0.06 a minute without the use of any Algebra. Once this is done, students understand the abstraction of: $0.10x + 5 = 0.06x + 8$.

- **Accessing and Applying Prior Knowledge to New Learning:** Determining specific, previously-learned skills necessary to learn a new concept and strategically organizing said skills in questions that will aid instruction.

- **Questions, Cues, and Advance Organizers:** Using a range of higher-order questions to preview new mathematics concepts and skills, help students extend and refine new learning, and scaffold student learning toward growing levels of transfer. Cues include formal and informal strategies to reinforce students' understanding of what they are learning and why they are learning it. Advance Organizer questions prepared prior to a lesson help guide students to understanding a topic. Advance Organizers (Ausubel) include outlines, flow charts, bulleted summaries, and visual representations that preview a lesson or unit or provide a guide for students, helping them to build a conceptual understanding of how ideas, concepts, and processes are interrelated.

- **Models to Develop Concepts and Skills:** Tools that aid students in developing a skill or concept (e.g., area models for multiplying binomials in lieu of F.O.I.L.).

Although models are abstract representations, they provide a conceptual schema to help students internalize patterns and connections.

- **Concept Development:** Strategies and processes that engage students in experience-based inquiry and investigation, reinforcing students' construction of cognitive schema and meaning—rather than just presenting information that they are required to recite or repeat to the instructor with no evidence of understanding or internalization.

- **Connections to Real-Life:** Basing learning in tasks relevant to the learner and aligned with his or her life experiences.

- **Games to Develop Concepts and Skills:** A means of motivating students to practice skills or develop conceptual understanding. Games generally involve some form of informal competition—and engage students in retaining key information and procedural knowledge.

Meta-Strategies for Number and Number Sense

Strategies Related to Numbers and Counting

As students develop a concept of counting as part of developing increasingly sophisticated number sense, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to counting; (2) constructing meaning, connecting new learning about the counting process to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can count and respond to counting-based problems with a level of independence and automatic use.

The following suggested counting-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Numbers and Counting

- **Concept Development:** Show a card with five to nine dots in a row so that they can be easily counted. Ask the child to count the dots. If the count is accurate, ask, "How many dots are on the card?" Many children will count again. One indication of understanding the first count will be a response that reflects the first count without recounting.

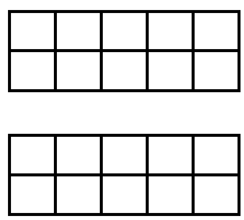
- **Games to Develop Concepts and Skills:** Engage students in games that involve counting. For example, this game for two students requires a deck of cards, a paper cup, and some counters. The first player turns over the top number card and places the indicated number of counters in the cup. The card is placed next to the cup as a reminder of how many are there. The second student rolls the die and places that many counter next to the cup. Together they decide how many counters in all. A record sheet with columns for “In the first cup” “In the second cup” and “In all” is used.

<u>Constructing Meaning and Correcting Errors and Misconceptions Related to Numbers and Counting</u>

- **Models to Develop Concepts and Skills:**

    1. Use ten frames to assist students in building a relationship of 10 as an anchor for numbers with the rule of: *Always fill the top row with counters first, starting on the left, the same way that you read. When the top row is full, counters can be placed in the bottom row, also from the left.*

    2. Call out random numbers between 0 and 10. After each number, the children change their ten-frames to show the new number. Students can play this game independently by having a list of about 15 “crazy mixed-up numbers.” One child plays the teacher and the other uses the ten-frame. An extension would be for children to make up their own number lists.



- **Investigation and Inquiry-Based Learning:** Engage students in finding pairs of numbers that add up to 100. As an extra challenge, have them find three numbers that add up to 100, four numbers, etc.

- **Manipulatives:** Have students measure the length of various objects (e.g., the length of the classroom, the height of a door, and the length of a whiteboard) using unifix cubes. Ask the students to determine the total number of cubes in the measurement.

<u>Promoting Guided and Independent Transfer of Big Ideas and Processes Associated with Numbers and Counting</u>

- **Investigation and Inquiry:** Give students a number and have them generate the ways that this number is used in real-life situations. For example, given the number 25, what are the different ways that 25 can be represented? Have the students create a *Book of 25* that captures the multiple expressions and representations (e.g., 25 cents, 25 %, ¼ of a dollar, and 25 past 12 on the hour). The students should work in pairs to share their books at the end of the activity.

The Base-Ten Number System

As students develop a concept of base-ten as part of developing increasingly sophisticated concept of place value, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to base ten; (2) constructing meaning, connecting new learning about the place value concept to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer.

When students can transfer what they have learned, they can think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they respond to the order of the counting numbers and their relative magnitudes with a level of independence and automatic use.

The following suggested counting-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to the Base-10 Number System

- **Manipulatives:** Play "Race to 100" on a place value mat. Students take turns rolling a die and adding that number of counters to a place value mat making the appropriate trades. The partner who reaches 100 first wins.

- **Investigation and Inquiry Based Learning:** Introduce the base ten blocks as a unit, a long, and a flat. To begin, assign the number 1 to the unit, 10 to the long, and 100 to the flat. Next, adjust the values so that 1 is now the long and 10 is the flat. What value does the unit block have? Continue this progression with the 1 next being the flat. What value do the long and the unit block have?

Constructing Meaning and Correcting Errors and Misconceptions Related to the Base-10 Number System

- **Manipulatives:** Have students measure the length of various objects (e.g., the length of the classroom, the height of a door, and the length of a whiteboard) using unifix cubes. Ask the students to determine the total number of cubes in the measurement by making and counting groups of ten.

- **Concept Development:** Extend students' knowledge of basic facts and the ten-structure of the number system so that counting is not required when adding and subtracting numbers. Encourage students to use the distance up or down to the multiple of ten to decompose a number, e.g., for 58 + 6, a student would change the 58 to a 60 and the 6 to a 4 which results in 60 + 4 is 64.

Promoting Guided and Independent Transfer of Big Ideas and Processes Related to the Base-10 Number System

- **Connections to Real Life:** Engage students in a project about real life examples of using groupings of tens. Provide the students with different photos of large gatherings of people, e.g., baseball stadium, a political gathering, and an Olympics' audience. Have students research the methods used to determine the number of people in large groups. For a final assignment, the students present their findings to the class for whole group discussion.

Comparison

As students develop a concept of ordering and comparing numbers as part of developing increasingly sophisticated number sense, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to ordering and comparing; (2) constructing meaning, connecting new learning about the comparison process to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can order and compare numbers and respond to comparison-based problems with a level of independence and automatic use. The following suggested comparison-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Comparison

- **Compare and Contrast:** Engage students in exploring sets of objects in order to develop a concept of greater than and less than. For example, students take turns rolling a die to determine the number of unifix cubes to stack. At the end of each round, the students spin a more or less spinner to see who wins the round. The partner who collects a total of 50 or more first wins.

- **Manipulatives:** Engage the students with the concept of fractional parts. Have students build paper fractions strips beginning with one half, and then extending to folding to ¼, 1/8 and 1/16. Class discussions should involve questions such as Can you find the answer to ¼ + 1/4? What about ¼ + ½?

- **Compare and Contrast:** List a set of unit fractions such as 1/3, 1/8, 1/5, and 1/10. Ask the students to put the fractions in order from least to greatest on a number line. Challenge the students to defend the way they ordered the fractions by explaining their thinking by using models.

Constructing Meaning and Correcting Errors and Misconceptions Related to Comparison

- **Games to Develop Concepts and Skills:** Create activities to build an understanding of the concept of integers. For example, play "Integer Football" where the desk is the football field of a vertical number line with one goal at +50 and the other goal is at -50. The students flick the paper football from the edge of the desk across the desk. Gains and loses are like positive and negative quantities. A positive team moves toward the positive goal and a negative team moves toward a negative goal. If the negative team starts on the -15 yard line and has a loss of 20 yards, then it will be on the +5 yard line.

- **Investigation and Inquiry-Based Learning:** Engage students in using benchmarks to easily tell which of two fractions is larger. Establish the benchmarks as 0, ½, and 1. Use fractions less than 1 to compare the benchmarks. For example, 3/20 is small, close to 0, where as ¾ is between ½ and 1. The fraction 9/10 is quite close to 1.

Promoting Guided and Independent Transfer of Big Ideas and Processes Related to Comparison

- **Connections to Real Life:** Have the students make up a home budget with a cash flow of $4,000 dollars a month showing debits and credits along the way for expenses for a 4 member family. As a final project, the students present to the class a comparison of the expenses and make a projection for a year long budget.

Equivalence

As students develop a concept of equivalence as part of developing increasingly sophisticated understanding of simplifying expressions, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to equivalence; (2) constructing meaning, connecting new learning about equivalence to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can use the properties of operations to rewrite expressions in equivalent forms and respond to equivalent problems with a level of independence and automatic use. The following suggested equivalent-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Equivalence

- **Models to Develop Concepts and Skills:** Use fraction models to build equivalent fractions. Have students use fraction manipulatives such as fraction bars to build sets of equivalent fractions.

- **Concept Development:** Use a fraction chart (below) to assist in building sets of equivalent fractions.

<u>Constructing Meaning and Correcting Errors and Misconceptions Related to Equivalence</u>

- **Investigation and Inquiry-Based Learning:** Engage students in finding the missing number in fraction equivalences. Give students an equation expressing equivalence between two fractions but with one of the numbers missing. The missing number can be the numerator or denominator. The task is to find the missing number and explain your solution. An added challenge is to express the fractions in lowest terms.

- **Multiple Representations:** Create a worksheet using a portion of either an isometric or rectangular dot grid paper. On the grid, draw an outline of a region and designate it as one whole. Draw a part of the region within the whole. The task is to name the fractional parts of the whole that they draw on the grid. The larger the size of the whole, the more fractional parts the activity will generate. As a final task, the students should find equivalent fractional parts in the shape.

<u>Promoting Guided and Independent Transfer of Big Ideas and Processes Related to Equivalence</u>

- **Connections to Real Life:** Have the students create a real-life word problem that involves equivalence between two fractional parts. For example, ask “How would you share 10 brownies with 4 students? 5 brownies with 2 students? Are these equal or equivalent relationships? Explain your responses.” They should create a picture book with illustrations of the equivalent fractions. After the book is completed, the students should read their book to a class of lower grade students.

Computation and Estimation

As students develop a concept of computation as part of developing increasingly sophisticated number sense, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to computation; (2) constructing meaning, connecting new learning about the computation process to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can compute flexibly, efficiently, and accurately and respond to computation-based problems with a level of independence and automatic use. The following suggested computation-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Associated with Computation and Estimation

- **Multiple Representations:** Compare concrete and real-word situations that associate fact families and equivalences. For example, students explore addition and subtraction as a pair of operations (12 – 8 = 4, 12 – 4 = 8, 4 + 8 = 12, 8 + 4 = 12) and multiplication and division as a pair. At a higher level, engage students in discussions that algebraic expressions can be named in an infinite number of different but equivalent ways (e.g. 5(k - 4) = 5k - 20 = 5x - (24 - 4)).
- **Non-Linguistic Representations:** Engage students in visualizing and drawing pictures of the basic facts to illustrate properties. Facilitate students making personal meaning of the properties and making connections with real life situations (e.g., 4 +3 = 3+ 4 which can be illustrated by a stack of unifix cubes with 4 red and 3 blue when rotated is the same as 3 blue and 4 red). Some students call this the "switch around rule."

Accessing and Applying Prior Knowledge to New Learning Associated with Computation and Estimation

- **Ball-Park Estimates:** Make ball-park estimates with various units and discuss strategies in order to maintain student's common sense approach to doing mathematics.

- **Substitutions:** Discuss how numerical calculations can be approximated by replacing numbers with other numbers that are close and easy to compute mentally.

Constructing Meaning and Correcting Errors and Misconceptions Associated with Computation and Estimation

- **Comparing and Contrasting:**

  1. Decide if/how the same number sentence or phrase (expression) can be associated with different situations and different number sentences or phrases (expressions) can be associated with the same situation. For example, multiplying by *x* is the inverse of dividing by *x*, any division calculation can be solved using multiplication. How are the representations similar and how are they different?

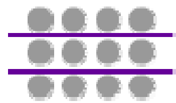
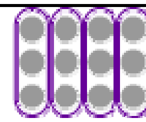
  2. Create graphic organizers to make connections among different algorithms through exploration of algorithms. For example, how does the standard division algorithm compare to partial quotients division?

  3. Compare and contrast benchmark fractions, decimals and percents and create a graphic organizer of benchmark fractions, decimals, and percents.

- **Multiple Representations:** Create examples that illustrate number relationships and properties (e.g., commutative) about decomposing numbers, for example, 5 is 3 plus 2, and also 2 plus 3, and -6 plus 11 as well as 11 plus -6.
- **Accessing and Applying Prior Knowledge to New Learning:** Use contextual problems with an emphasis on words, pictures, and numbers. Students use words and/or writing to express what they did and why it makes sense related to what they already know. Translate and simplify expressions using different symbols, e.g., 12 ÷ 2 is the same as 12/2.
- **Connections to Real-Life:** Draw representations and use manipulatives to visualize partitive and quotative division models:

  1. Partitive model: Theresa had 12 apples and 3 bags. How many apples can she put in each bag?

  2. Quotative model: Molly had 12 apples and several bags. If she puts 3 apples in each bag, how many bags did she use?

	Partitive Division	Quotative Division
Equation	12 ÷ 3 = 4	12 ÷ 3 = 4
Action	Partition into 3 groups.	Repeatedly subtract 3.
Result	There are 4 in each group.	There are 4 groups of 3 in 12.
Question	Twelve apples, 3 bags -- how many in each?	Twelve apples, 3 in a bag -- how many bags?
Model		

Promoting Guided and Independent Transfer of Big Ideas and Processes Associated with Computation and Estimation

- **Connections to Real-Life:** Engage students in a real-life situation where the ongoing report on sales is imperative to a successful company. For example, give students a role as a real-life manager of a toy production company, Willard Sales, who has decided to consider adding a new line of toys named widgets to their production inventory.

  As the manager, you begin by buying 275 widgets wholesale for $3.69 each. In the first month, the company sold 205 widgets at $4.99 each. How much did Willard make or lose on the widgets? Do you think that Willard Sales should continue to sell widgets? Prepare a report to the company's board of supervisors giving a rationale for the next steps. You will be presenting your report to the class who will be your board of supervisors (Van de Walle, 2007).

Geometry

As students develop a concept of geometry as part of developing increasingly sophisticated spatial sense, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to shapes and space; (2) constructing meaning, connecting new learning about geometric concepts to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can count and respond to geometric-based problems with a level of independence and automatic use. The following suggested geometric-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Geometry

- **Accessing and Applying Prior Knowledge to New Learning:** Engage students in comparing triangles to determine similarities and differences. For example, have students draw five different triangles. Have students explain why they think each triangle is different by comparing the angles and sides (Van De Walle, 2004).

- **Comparing and Contrasting:** Engage students in comparing and contrasting two-dimensional shapes in order to categorize them based on their attributes. For example, students can work in groups of four with a set of plane figures to:

    1. Describe the attributes about their shapes;
    2. Compare and contrast their shapes;
    3. Identify shapes that have the same attributes as a target shape (e.g. "This shape is like the target shape because it has a curved side and a straight side.");
    4. Sort the shapes based on their attributes and explain the rules of the sort; and
    5. Complete a "secret sort" where a collection of shapes share a common attribute and the other students have to find other shapes that fit that attribute and/or guess the "secret" attribute. (Van De Walle, 2004)

Constructing Meaning and Correcting Errors and Misconceptions Related to Geometry

- **Investigation and Inquiry-Based Learning:** Engage students in the investigation of transformations and tessellations using trapezoids, rhombi, or chevrons in order to understand that congruent figures remain congruent through transformations, including reflections, translations, and rotations. Have students consider the shapes as "tiles" for a mosaic or floor. Have students create a "regular tiling pattern" with two properties: (1) the tiles must fit together without overlapping and without leaving any spaces and (2) the design is to be the same continuously, notwithstanding the size of the space. Several different tiling patterns are possible for each of the three tile choices. Isometric grid paper can be used for this. (Van DeWalle, 2004)

- **Games to Develop Concepts and Skills:** Engage students in playing the "Hidden Positions" game in order to understand that the location of points or objects on a plane can be described using a coordinate system. (To create the game boards, draw an 8-inch square on cardstock. Subdivide the squares into a 3 x 3 inch grid.) Pair two students and have them sit with a "screen" separating their desktop space so that neither student can see the other's grid. Each student will have four different pattern blocks. The first player places a block on four different sections of the grid. He/she then tells the other player where to put blocks on her grid to match his/her own. When all four pieces are positioned, the two grids are checked to see if they are alike. Then the players switch roles. The first time the game is played, the teacher should model the game once by taking on the role of the first student. Students should use relative position words such as *top row, middle, left, and right* (Van De Walle, 2004).

- **Investigation and Inquiry-Based Learning:** Engage students in measuring the diameter, circumference, and radius of circles (e.g., cylindrical containers, lids, hula hoops, clock faces, bicycle tires, and other common circular objects) in order to approximate the value of pi through the relationship between the diameter and circumference. Students can record their information in a table and use a calculator to complete their calculations.

Promoting Guided and Independent Transfer of Big Ideas and Processes Related to Geometry

- **Comparing and Contrasting:** Engage students in the recognition, description, drawing, classification, and comparison of geometric figures in order to understand that plane and space figures can be described based on their core attributes and that real world situations can be used to think about these attributes. Students will analyze a variety of polygons and their core attributes.

    1. Materials needed are several books about polygons so that students choose a tier based on their interest in a particular book. Three such books are *The Greedy Triangle*, *Shape Up,* and *A Cloak for the Dreamer*.

    2. This lesson could also be differentiated according readiness by varying the difficulty of the books. Each student will read his/her book and create his/her own short picture book in which they describe, draw, classify, and compare polygons and parts of the polygons. As a culminating activity, the studentsread their books to other students.

Measurement

As students develop a concept of measurement as part of developing increasingly sophisticated number sense, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to measuring; (2) constructing meaning, connecting new learning about the measurement process to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can measure and respond to measurement-based problems with a level of independence and automatic use. The following suggested measurement-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Measurement

- **Concept Development:** Engage students in the use of non-standard measurement (e.g., kindergartners use paper clips or strips of paper to measure things)—moving students toward an understanding of the need for common measurement tools and systems.

- **Connections to Real-Life:** Students develop "their own personal benchmarks or references for estimating measures (e.g., a 12 inch ruler is about the length of a standard sheet of paper, a paper clip is about the weight of 1 gram).

- **Concept Development:** Introduce the concept of proportionality by having students find the number of paper clips equal to the length of their foot and then have them find the number of paper clips for two of their feet.

- **Investigation and Inquiry-Based Learning:** Provide students with five different versions of a ruler (e.g., one that has zero at the edge, one that has zero at an indent, and several broken rulers with the beginning of the ruler broken off). Students will measure the length of a picture of an object using all of the rulers. They then will compare their answers and explain in writing how they are the same or different. Through class discussion, they should come to an understanding of why it is important to distinguish a starting point and ending point when measuring an object.

Constructing Meaning and Correcting Errors and Misconceptions Related to Measurement

- **Accessing and Applying Prior Knowledge to New Learning:** Engage students in estimating prior to every measurement in order to maintain student's common sense approach to doing mathematics. On an ongoing basis, students should estimate solutions to a given measurement or problem. For example, if a paper clip is 2.8 cm long and we want to know how many we can line up along a 30 cm book, one strategy students could use would be to round up the length of the paper clip to 3 cm and divide 30 cm by 3 cm.

- **Investigation and Inquiry-Based Learning:** Set up measurement activities in which students are engaged in investigating, estimating, and measuring real things. Allow students to choose the measurement tool or unit they use to measure. Ask them to explain their choices and the process they use. After working with a partner, the team should come to a consensus of the correct answer.

Promoting Guided and Independent Transfer of Big Ideas and Processes Related to Measurement

- **Comparing and Contrasting:** Engage students in comparing and contrasting various units (e.g., inches to centimeters, pounds to "stones") when measuring items such as football fields, the weight of a car, and the height of a building. Have the class agree on a real world situation to investigate. Pairs of students then measure the item and then express the measure in multiples units (e.g., yards, feet, or meters). Students use technology to check their answers. At the end of the activity, students share their results with their class.

Probability and Statistics

Data and Graphs:

As students develop a concept of data collection as part of developing increasingly sophisticated understanding of representing and interpreting data, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to data analysis; (2) constructing meaning, connecting new learning about the data collection process to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can collect, represent and analyze data, and respond to data-based problems with a level of independence and automatic use. The following suggested data-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Data and Graphs

- **Connections between Concrete and Symbolic:** Begin with concrete/real graphs to picture graphs and move toward symbolic representations.

- **Investigation and Inquiry-Based Learning:** Graph a daily survey question of the day that is relevant to students' lives. The purpose for collecting data and analyzing a graph is to answer a question. Attendance graphs allow students to experience different types of representations (e.g., tally marks, bar graph, Venn diagram, line plot, etc.)

Constructing Meaning and Correcting Errors and Misconceptions Related to Data and Graphs

- **Multiple Representations:** Create different graphs from the same data to illustrate how different information can be learned.

1. Allow opportunities for students to collect and organize numerical and categorical data using graphs, tables and charts. For example, collect student's heights and discuss how to select an appropriate representation for the type of data (e.g., circle graph for part to whole data, histogram to show continuous numerical data, etc.).

2. Create graphs that are centered about the class, school, or community. Later students can begin to develop questions that require them to collect data outside of the classroom and can be related to other areas of study.

Promoting Guided and Independent Transfer of Big Ideas and Processes Related to Data and Graphs

- **Investigation and Inquiry-Based Learning:** Have students conduct a survey requiring them to represent their findings in three different graphical forms. Students present their graphs and, as a class, make observations and inferences about data from their survey. The important question when analyzing a graph is, "What information do we learn from the graph?" Discuss as a class how the graph helps to analyze the question posed.

Probability

As students develop a concept of chance as part of developing increasingly sophisticated understanding of probability, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to probability; (2) constructing meaning, connecting new learning about the probability process to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can determine the chance of an event occurring and respond to probability-based problems with a level of independence and automatic use. The following suggested probability-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Probability

- **Manipulatives:** Engage students in different types of experiments using coins, spinners, dice, tiles, and other tools where the sample space and probability of each outcome can be determined.

- **Models to Support Concepts and Skills:** Use probability meter (fraction, decimal and percent) to discuss likelihood of an event. Begin with words like certain, impossible and possible and develop the varying degrees of these words (unlikely, equally likely, and very likely). Children will begin to develop that the probability of an event are on a continuum.

Constructing Meaning and Correcting
Errors and Misconceptions Related to Probability

- **Comparing and Contrasting:** Engage students in discussions about why particular outcomes are more likely than others. Through exploration and class discussion, students will begin to develop an understanding that particular outcomes are more likely to occur regardless of luck.

- **Games to Develop Concepts and Skills:** Use games of chance to compare experimental and theoretical probability.

Promoting Guided and Independent Transfer of
Big Ideas and Processes Related to Probability

- **Comparing and Contrasting:** Create a carnival game of chance that is rooted in experimental and theoretical probability. Students invent the rules, based on chance, for the player(s) to win. Is the game fair or unfair? Would people be willing to play? What elements of chance entice players? Student games are evaluated based on comparisons to real-life games. Students share their carnival game at the end of the activity.

Patterns, Functions, and Algebra

As students develop a concept of equivalence as part of developing increasingly sophisticated understanding of simplifying expressions, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to equivalence; (2) constructing meaning, connecting new learning about the counting process to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can use the properties of operations to rewrite expressions in equivalent forms and respond to equivalent problems with a level of independence and automatic use. The following suggested equivalent-related meta-strategies are organized along this learning continuum.

Part I: Developing Beginning Understandings of Properties

Acquiring Key Concepts and Skills
Related to Patterns, Functions, and Algebra

- **Non-Linguistic Representations:** Model equations by developing the idea of balance. Students can develop rules for the balance based on experimentation: will adding 3 to both sides of the equation change the way the equation looks? Will it change the value of x? If x always equals 4, what other equations can be written?

Constructing Meaning and Correcting
Errors and Misconceptions Related to Patterns, Functions, and Algebra

- **Manipulatives:** Measure objects using different standard (inches, cm, mm, yards, etc.) and non standard (hands, sheets of paper, textbooks) units of measurement. Generalize that different measures are equivalent because the item being measured never changes, and that we know different units of measurement such as 1 inch = 2.54 cm.

Promoting Guided and Independent Transfer of Big Ideas and Processes
Related to Patterns, Functions, and Algebra

- **Investigation and Inquiry-Based Learning:** Create a number project in which student generate as many different mathematical representations of a given set of number between 1 - 100. For example, 12 can be represented as 3×4, 1×2, 2×6, there are twelve numbers on the clock, a dozen (an equivalent word for 12) eggs usually comes in two rows of 6, 12 minutes are equal to 720 seconds and of an hour, etc. Student make a book of their number representations that they share with the class for whole group discussions.

Part II: Developing Advanced Concepts of Properties

As students develop an understanding of properties as part of developing increasingly sophisticated sense of algebra, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to properties; (2) constructing meaning, connecting new learning about the rules of arithmetic and algebra to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they use the properties flexibly, efficiently and accurately, and respond to expression and equation-based problems with a level of independence and automatic use.The following suggested property-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Understanding Properties

- **Manipulatives:** Explore the properties by modeling simple arithmetic or algebraic expressions using manipulatives by searching for equivalence in the expressions. Develop rules for the four operations relative to order (commutative), grouping (associative), break-apart (distributive) rules.

Constructing Meaning and Correcting
Errors and Misconceptions Related to Understanding Properties

- **Comparing and Contrasting:** Compare and contrast the values of operations for frequent errors related to the properties. How can we prove or disprove the equivalence of 4 + 3 + 2 = 10 – 1? How are the quantities on both sides of the equal sign the same and different?

- **Multiple Representations:** Model the distributive property using partial products. Initially using numbers such as 5(24) by writing it as 5(20 + 4) and creating an area model showing the area of, and. This can be extended to variables by drawing the model as an array and checking for equivalency by substituting values in for the variable.

Promoting Guided and Independent Transfer of
Big Ideas and Processes Related to Understanding Properties

- **Making Connections to Real-Life:** Students create real-life examples of the identity properties. An example of the identity property of addition can be found in a spreadsheet balancing withdrawals and deposits made in an account. When the account shows a day to day summary, many days may have $0 added to the account. The zero is an essential element to tell the account holder a valuable piece of information, while mathematically not changing the balance. These real-life examples are created over the unit of study. At a final project, the students create a Word document of their examples that they will present to a peer-partner.

Algebraic Patterns:

As students develop a concept of patterns as part of developing increasingly sophisticated sense of algebra, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to patterns; (2) constructing meaning, connecting new learning about the patterning process to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can use patterns to make predications and generate rules and respond to pattern-based problems with a level of independence and automatic use. The following suggested pattern-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Algebraic Patterns

- **Manipulatives:** Model growing patterns using square tiles, snap cubes to display the pattern and to use as a reference when describing the way the student can describe how the pattern grows and predict the unknown terms. For example, if the pattern 2, 5, 8, 11... were modeled with cubes, what would be the 10^{th} term? The 100^{th}? The n^{th}?

Constructing Meaning and Correcting
Errors and Misconceptions Related to Algebraic Patterns

- **Multiple Representations:** Explore a visual model of growing patterns to create expressions for how to "see" the n^{th} pattern. Students can compare different explanations of growth, written as an algebraic expression or in words.

Promoting Guided and Independent Transfer of
Big Ideas and Processes Related to Algebraic Patterns

- **Making Connections to Real-Life:** Research an aspect of nature or every day life to collect data that the student suspects might behave in an expected way. The student can use the pattern to develop a model that describes the behavior algebraically. For example, investigate how text messaging rates can be described algebraically. Student present their data in multiples ways and presents their finding to the class at the end of the unit of study.

Reinforcing Key Concepts and Big Ideas Related to Algebra

As students develop a concept of variable as part of developing increasingly sophisticated understanding of algebra, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to algebra; (2) constructing meaning, connecting new learning about the algebra to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can use variables flexibly and respond to algebra-based problems with a level of independence and automatic use. The following suggested variable-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Algebra

- **Accessing and Applying Prior Knowledge to New Learning:** Develop a list of vocabulary words that can be *clues - not rules* - to connect mathematical operations to verbal phrases and/or terms. Context needs to be considered as "rules" for some words (ex 'more' as compared to 'more than') are not exclusively used for one operation.

Constructing Meaning and Correcting Errors and Misconceptions Related to Algebra

- **Accountable Talk:** Make generalizations about why we use universal variables (such as r for the radius of a circle), and how an equation with variables can work with infinite inputs to give the corresponding output.

- **Investigation and Inquiry-Based Learning:** Explore the relationship between the diameter of a circle and the circle's circumference by recording these measurements and searching for a pattern. This pattern can be described in words and by using a variable equation. Students will conclude that diameter divided by circumference results in the universal constant, pi.

Promoting Guided and Independent Transfer of Big Ideas and Processes Related to Algebra

- **Making Connections to Real-Life:** Create sentences with real world, relevant scenarios and then write them using algebra. For example: t = 3h: The total snowfall in the big snow storm could be found by multiplying 3 inches for each hour it snowed. Students collect ten of these story/algebra examples to share with a peer-partner.

PROPORTIONALITY:

As students develop a concept of proportionality as part of developing increasingly sophisticated understanding of linear functions, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to proportions; (2) constructing meaning, connecting new learning about proportional relationships to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can use proportions flexibly and efficiently, and respond to proportions-based problems with a level of independence and automatic use. The following suggested proportion-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Related to Proportionality

- **Non-Linguistic Representations:** Create ratio tables that show several equivalent ratios. Ratio tables can be used as a means to find proportionate quantities. Numbers in ratio tables can increase by a common multiple, and as students become more familiar with them, can increase by proportionate values that make sense to the student.

Constructing Meaning and Correcting
Errors and Misconceptions Related to Proportionality

- **Comparing and Contrasting:**

    1. Plot points on the ratio table onto a graph and compare and contrast the two representations. For example, how would a set of points in a table be more or less useful than if they were represented in a graph?

    2. Investigate the 'doubling effect' on direct variation (equations in the form of or) and on equations that have a non-zero constant. Are the points (2, 3) and (20, 30) always on the same line?

Promoting Guided and Independent Transfer of Big Ideas and Processes Related to Proportionality

- **Connections to Real-Life:** Create proportionate situations to analyze. What questions can be asked based on this situation? For example, three pounds of turkey must be cooked for 1 hour. Students can experiment with different functions representing a ratio. They can make generalizations that the solutions to proportions can be found on the graph (on the line), or can be found by proportional reasoning and plotted on a graph to form a straight line. A line in direct variation on a rectangular coordinate system (Cartesian Coordinate Plane) can be used to create an infinite number of proportional rectangles. Choosing a point on the line and drawing a vertical line to the x-axis and a horizontal to the y-axis would result in a rectangle. The corresponding points on the two axes could be related back to a situation such as the turkey problem (one hour for each three pounds). A line of direct variation would have points corresponding with (2 hours, 6 lbs), (3 hours, 9 lbs.), (3.5 hours, 10.5 lbs.) on the x- and y-axes respectively. Students finalize the project by creating a powerpoint presentation to present to the class.

RELATIONS AND FUNCTIONS:

As students develop a concept of proportionality as part of developing increasingly sophisticated understanding of linear functions, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to relations and functions; (2) constructing meaning, connecting new learning about relations and functions to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can use relatons and functions flexibly and efficiently. The following suggested relations- and functions-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Associated with Relations and Functions

- **Models:** Use graphing calculators to integrate tables, graphs, and equations.

- **Non-Linguistic Representations:** Visualize a function machine. Students can imagine a machine that takes one number (input) and changes it into another number (output.) Students can determine the rule, or relationship between the input and output.

- **Games to Develop Concepts and Skills:** Play "Guess My Rule." With partners, student A can see the relationship between input and output on a card. Student B makes up any number as the input. Student A uses the relationship on the card to determine the input. This is repeated until Student B is able to determine the relationship. Then players switch roles.

Constructing Meaning and Correcting Errors and Misconceptions Associated with Relations and Functions

- **Making Connections Between Concrete and Symbolic:** Connect word problems, tables, graphs, and equations by making sense of one and then describing the usefulness of the other representations. Include all representations together to provide a full context to the math concept - the word problems, tables, graphs, and equations.

- **Models:** Use a vertical line test to determine if something is a function and tie it back to input-output values. Graph a function and determine if any two points comprising the graph of the function are lined-up vertically. Discuss with students the meaning of two vertically aligned points in relation to the actual function – "Should there be different output values when the same input value is placed in the function machine?"

- **Multiple Representations:** Observe changes in a functions behavior using a graphing calculator. Compare the function's base form to a form involving more complex parameters. For example, how do the graph and table of change when the coefficient goes from one to two and the equation is? Students can use the calculators to determine how changing parameters effects slope, translations in the x- and y- directions, and reflections.

Promoting Guided and Independent Transfer of Big Ideas and Processes Related to Relations and Functions

- **Comparing and Contrasting:** Use the rule of four to compare situations (such as cell phone plans) in order to evaluate which relationship is best for the given situation. Graphs, equations, and tables can be used as supportive evidence. Students structure their notebooks to illustrate the rule of four over a unit of study. They share their notebook at the end of the unit with a peer for review. A rubric is used to evaluate the notebook.

- **Comparing and Contrasting:** Create situations from real-life that can be represented as functions in the form of y = ax to make observations, descriptions and generalizations about the function. Students should experiment with *a* as a positive whole number, negative whole number, positive rational number, and negative rational number. Analyze how the differences in the algebraic equation may affect the outcome in the real-life situation. Students publish their results in a booklet format to be reviewed by peers at the end of the unit.

EQUATIONS AND INEQUALITIES:

As students develop a concept of proportionality as part of developing increasingly sophisticated understanding of linear functions, they will move back and forth between and among three predictable stages of learning: (1) initial acquisition of key concepts and skills related to equations and inequalities; (2) constructing meaning, connecting new learning about equations and inequalities to prior experiences and understandings; and (3) eventual movement toward guided and independent transfer. When students can transfer what they have learned, they can use equations and inequalities flexibly and efficiently. The following suggested equations- and inequalities-related meta-strategies are organized along this learning continuum.

Acquiring Key Concepts and Skills Associated with Equations and Inequalities

- **Concept Development:** Model equations by developing the idea of balance. Students can develop rules for the balance based on experimentation: will adding 3 to both sides tip the balance? Will doubling the quantities on both sides tip the balance? Answers can be confirmed by numeric trials and proven with algebraic generalizations.

Constructing Meaning and Correcting Errors and Misconceptions Associated with Equations and Inequalities

- **Comparing and Contrasting:** Substitute values for variables and evaluate the equation to determine if the substituted value actually results in equal values. Will the equation be true if x is 2? If x is 3?

- **Multiple Representations:** Use a graphing calculator to input and graph the equation. Students can use the TRACE feature or scroll to points on the line to identify the ordered pair.

- **Making Connections to Real-Life:** Develop word problems that can be modeled with the multiplication or division of a negative number which changes the direction of the inequality sign. Using word problems where the context necessitates a positive coefficient and a negative coefficient to support students' understanding of this "changing direction" rule. Using a number line students can see that similar contextual solutions are found when, and. Examples can include: Skydiving, bungee jumping, scuba diving, etc. Students can model the word problem to the number line. Ex: Your math class is playing a version of Jeopardy where each question is worth 5 points. If you answer a question wrong, your team earns -5 points. It is in the final round, and your opponent has -50 points. All that matters is that you win - even if you win with negative points. If you currently have zero points, how many questions could you answer incorrectly in a row, and still win? Model the following on a number line: -and. Both inequalities model the same scenario, the negatives are implied in the second (how many negative points can I earn and still have a score greater than -50, or how many questions can I get wrong and still have a score less than 50)

<u>Promoting Guided and Independent Transfer of</u>
<u>Big Ideas and Processes Related to Equations and Inequalities</u>

- **Success Criteria for Self- and Peer-Evaluation**: Develop equations and inequalities to be solved algebraically by justifying steps using notions of equivalence and the rules of arithmetic and algebra. Equations developed by one student can be solved and justified by a partner, and assessed together.

Teaching Literacy Strategies Within the Mathematics Classroom

1. Emphasize the importance of reading comprehension strategies as part of students' work with mathematical word problems (i.e., the need to "unpack" the meaning of text presented within the problem).

2. Encourage students to share insights and approaches to problem solving in one-on-one and small group debriefing sessions.

3. Ask students to summarize and synthesize what they have learned in a particular lesson (or lesson segment) using exit slips, reflective journal entries, and other forms of brief, informal writing activities in digital or print formats.

4. Have students present oral summaries and presentations of their approaches to mathematical problem solving (e.g., creating a power point or video, demonstration, recap).

5. Approach academic vocabulary in mathematics strategically, ensuring that students have a conceptual understanding of key terms and phrases.

6. Integrate technology into students' exploration of key mathematical concepts and problem-solving processes, including electronic opportunities to debrief, reflect, and revise thinking.

Learning Theory and Mathematics Achievement

Cognitive Learning Theory:

- There are no blank slates. We construct meaning by attaching new knowledge to existing schema.
- Students should be continually engaged in asking such questions as "Why are we learning this? How does this connect to me and my previous experiences? To what extent can I make connections with prior learning and the world beyond the classroom?"
- Learning is highly situated. Transfer does not necessarily occur naturally. It requires that students be coached to move from initial acquisition of new knowledge and skills toward growing levels of constructed meaning and independent transfer. (Scaffolding is key.)
- Learning often occurs in associational and recursive ways, not in neat, linear fashion. Students need time to reflect and express their understandings—as well as misunderstandings and areas in which they need help.
- Effective learning is strategic: Students need to learn when to use knowledge, how to adapt it, and how to self-assess and self-monitor.

Implications for Enhancing Students' Mathematics Development: ***Promoting students' mathematics development requires constant on-the-spot coaching and criterion-based feedback to learners so that they can revisit, revise, rethink, and refine their understanding of text, including life experiences. Such standards-based coaching helps them to play an active role in progressing from basic knowledge acquisition toward proficient and advanced levels of learning goal mastery.***

Constructivist Teaching and Learning:

- Students (and their varying readiness levels, interests, and learning profiles) should be at the heart of the teaching-learning process.

- The teacher should be a facilitator and coach, not just a dispenser of information.

- Content should be presented whole to part, with emphasis upon big ideas and essential questions so that every learner can understand the "big picture."

- Assessment and instruction should be seamless, with continuing feedback to learners about how they are doing and how they can improve their learning process.

- Experiential learning, inquiry, and exploration supersede lecture and "transmission" of information.

Implications for Enhancing Students' Mathematics Development: ***Effective mathematics instruction places the learner at the center of his or her own assessment and learning process. It actively engages students in the processes of self-monitoring, self-regulation, self-adjustment, and metacognition. Through it, students see the "big picture": i.e., Why are we learning this? How can I help myself to improve? How can I enhance my understanding of text—and my ability to communicate how I am experiencing things—and constructing meaning?***

Brain-Compatible Teaching and Learning:

- The brain asks "Why?" Therefore, the "compelling why" should be addressed at the beginning of every lesson or lesson segment.

- The brain searches for connections, associations, and patterns: Students need help to see patterns, connections, and relationships within what they are learning.

- The memory system to which we most often teach (the declarative/semantic/linguistic) is inferior to the episodic (i.e., the memory of emotion, relationships, and narrative) and procedural memory (i.e., the physical/muscular/tactual-kinesthetic) systems in storing and retaining knowledge.

- The brain "downshifts" when it perceives threat in the environment. Classrooms must be safe and inviting learning communities.

Implications for Enhancing Students' Mathematics Development: ***Brain-compatible teaching and learning reinforces the sense that "we are all in this together," including emphasis upon peer coaching, peer response groups, and peer celebration of learning progress. By engaging all memory systems, students retain more information, skills, and procedures—and deepen their understanding of the "compelling why…" The more engaging and physically active the learning experience, the more students will retain and understand.***

Learning Style Preferences, Learning Modalities, and Multiple Intelligences:

- We take in impressions and construct meaning about our world through multiple sensory channels and modalities.

- There is no single way to learn: We construct meaning, perceive our world, and make judgments based upon a variety of learning styles.

- According to Howard Gardner, intelligence is a *potential*, not an innate gift, and manifests through *multiple forms* such as the linguistic, logical/mathematical, visual/spatial, musical, bodily/ kinesthetic, interpersonal, intra-personal, and naturalist/ecological.

<u>Implications for Enhancing Students' Literacy Development</u>: ***Using a range of feedback tasks and strategies ensures that students' learning style preferences are accommodated. When students are periodically allowed options for demonstrating their achievement of learning goals (e.g., via performance tasks involving multiple modalities and formats, self-reflections, cooperative learning, and scaffolded prompts and projects), they activate a range of intelligences and skills. Additionally, great literacy instruction takes into account students' unique needs and strengths, including their varying readiness levels, interests, and learning profiles. The more students can "see themselves" in what they are doing, the greater their level of engagement and retention.***

<u>Emotional Intelligence</u>:

- Dan Goleman and the Stanford "marshmallow experiments": Emotional intelligence is a more powerful determinant of life success (e.g., relationships, career, schooling) than the cognitive/ intellectual.

- Students need coaching and support to develop a sense of efficacy and social consciousness.

- Classrooms should be safe and inviting communities of learning.

<u>Implications for Enhancing Students' Mathematics Development</u>: ***Interacting with rich and engaging math problems (print, electronic, visual and performing arts, life experiences) can provide students with powerful opportunities to develop emotional intelligence. The more students are empowered to be responsible for their own learning process, the greater their sense of efficacy and self-regulation. During effective mathematics instruction, students learn to monitor and adjust their own behavior in relationship to learning goals. Classroom climate is enhanced by expanding emphasis upon student interaction and direct engagement in the learning process.***

<u>Creativity and Flow</u>:

- Mihalyi Csikzentmihalyi: "Flow is a condition in which we experience a sense of timelessness, engagement, and stress-free challenge."

- Creativity requires the ability to free associate and brainstorm.

- We must help students to push the limits of their knowledge and ability.

- Students must be taught to tolerate and explore situations and ideas that are ambiguous and open-ended.

<u>Implications for Enhancing Students' Mathematics Development</u>: ***An expanded emphasis upon active engagement in metacognitive and self-regulating processes encourages students to overcome the perception that: "My teacher tells me how I am doing so I don't have to be responsible for doing it." It reinforces the "locus of control" as being centered within the learner. By engaging students in active self-assessment and self-monitoring, formative assessment expands the likelihood that students can gain acceptance of challenging and open-ended tasks and situations. Effective mathematics instruction encourages students to demonstrate creative self-expression and experience flow states (i.e., the universal sense of timeless engagement where tasks are sufficiently engaging and challenging to take students "outside" themselves).***

Research-Based Mathematics Instructional Strategies

Research-Based Mathematics Instructional Strategies					
Strategy	**What's That?**	**How do I find out more about it? Show me an Example!**	**Where does it fit in the instructional sequence?**		
			Pre	During	Post
Engaging Students by Helping Students Understand Tasks					
Reword Directions or Questions	Provide several opportunities for students to receive directions. Reword questions to support language acquisition.	http://www.educationatlas.com/solving-math-word-problems.html **Suggested Resources:** charts, document cameras	X	X	
Visual and Auditory Directions	Provide visual and auditory directions to support different learning styles.	http://homeworktips.about.com/od/homeworkhelp/a/visual.htm	X	X	
Preview Vocabulary	Use a series of thought- provoking statements before the math lesson to pre-teach vocabulary and build curiosity.	http://www.eduplace.com/state/pdf/author/chard_hmm05.pdf Student response systems ("clickers"), word processing, online surveys	X	X	
Highlight Key Information	Encourage students to highlight or underline important concepts and main ideas within a word problem.	http://www.metamath.com/lsweb/fourls.htm	X	X	
Examples of Finished Product	Provide examples of finished products to set the stage for completing high level tasks.	Concept mapping software, word processing, presentation software	X	X	X
Strategies for Helping Students Access Mathematics in Varied Ways					
Activate Prior Knowledge	Use questions and related activities to help students activate prior knowledge.	http://www.exploratorium.edu/IFI/resources/museumeducation/priorknowledge.html	X	X	
Make Connections Across Math Topics	Assist students in making connections to previously taught lessons and topics.	Discussion boards	X	X	

Move from Concrete to Represen-tational to Abstract	Moving from the concrete to the abstract assists students in building a deep understanding of the mathematics.	Real Life Problem > Graph or Table of Data > Function http://crlt.indiana.edu/research/csk.html www.mathalicious.com Document camera, concept mapping software, presentation software	X	X	X
Multiple Represen-tations	Use multiple representations such as word problems, equations/expressions, tables, graphs, and models during the problem solving process.	http://www.pictorialmath.com/updates_downloads/PresenterGuide.ppt **Rule of Five**	X	X	
Provide Additional Examples	Offer examples of finished projects so that students can build a vision of a high quality product.		X	X	X
Manipula-tives	Two- and three-dimensional models and representations, encouraging students to synthesize and make connections (e.g., a terminology collage, a big concept mobile)	http://www2.scholastic.com/browse/article.jsp?id=4003 www.ExploreLearning.com National Library of Virtual Manipulatives http://nlvm.usu.edu/	X	X	
Graphic Organizers and Concept Maps	A visual representation used to organize and show relationships between content	http://www.k8accesscenter.org/training_resources/mathgraphicorganizers.asp	X	X	X

Use Technology Strategies	Use document cameras, Smart technologies, clickers and other technologies to support student accessing and assessing the mathematics.	http://www.ncsu.edu/meridian/sum2003/math/index.html Discussion boards, blogs, wikis, word processing, presentation software	X	X	
Use Visuals – Charts or Projected Images	Use color coding on visuals to sort concepts or the big ideas.		X	X	
Alternative Ways for Students to Show What They Know	Provide more than one way for students to demonstrate their understanding.	http://www.virtualsalt.com/crebook4.htm Rule of Five Students discuss their ideas in groups	X	X	X
Kinesthetic Learning Opportun-ities	Use "hands on" student learning experiences, and field work outside the classroom.	http://www.metamath.com/lsweb/fourls.htm#tk	X	X	
Promoting Understanding Through Discourse Strategies					
Think-Pair-Share	Pair students for the problem solving process. Think-Pair-Share is a low-risk strategy to get **many** students actively involved in classes of any size.	http://clte.asu.edu/active/usingtps.pdf	X	X	
Cooperative Learning	Engaging students in solving problems with others promotes interacting and discussing multiple strategies.	http://ruby.fgcu.edu/courses/80337/Lourenco/CoopLearn/sld001.htm	X	X	
Use Questions, Prompts, and Hints	Use questions, prompts, and hints to scaffold the learning sequence for students.	http://condor.admin.ccny.cuny.edu/~group4/Van%20Der%20Stuyf/Van%20Der%20Stuyf%20Paper.doc Interactive whiteboard, word processing, presentation software	X	X	X

Higher-Order Questioning	Using a range of questioning strategies to promote student engagement and discussion (e.g., follow-up probes, application, analysis, synthesis, evaluation)	http://www.med.wright.edu/aa/facdev/_Files/PDFfiles/QuestionTemplates.pdf http://books.heinemann.com/math/construct.cfm	X	X	X
Timely and Construc-tive Feedback	On-going feedback on student's work provides students with a motivation to stay on task and to meet expectations.	http://en.wikipedia.org/wiki/Formative_assessment	X	X	X
Think Alouds	Teachers (and students) verbalize how they are engaged in a thinking process, including articulated steps in problem solving or decision making or skills applications	Think Alouds at AdLit.org (also has links to more resources) Document camera, interactive whiteboard, digital video recorder, Storyline Online	X	X	X
Summar-izing	Summarization gives the opportunity for students to show what they know after being engaged in a task.	http://scimath.unl.edu/MIM/files/research/EckmanS.pdf		X	X
JIGSAW	A cooperative learning process in which students form a base group, then move to an expert group, and then return to their base to teach base group members about what they learned as part of their expert group experience	http://www.jigsaw.org/ http://www.litandlearn.lpb.org/strategies/strat_jigsaw.pdf online discussion boards, blogs, wikis		X	

Instructional Strategies for Differentiation					
Adjust the Level of Difficulty	Present students who are struggling with a problem a different way of approaching the problem. In this case, provide multiples levels of difficulty to meet the needs of each and every student.	http://www.prufrock.com/client/client_pages/GCT_Readers/Math/Ch._4/Tiered_Lessons_for_Gifted_Children.cfm	X	X	X
Use Friendlier Numbers	Present students who are struggling with a problem a different way of approaching the problem. In this case, provide students with friendlier numbers (smaller numbers) to solve word problems.	Interactive whiteboards, concept mapping software, document camera, spreadsheet software	X	X	
Break Tasks into Smaller Parts	Present students who are struggling with a problem a different way of approaching the problem. In this case, provide students with the problem broken down into smaller parts.		X	X	
Adjust the Amount of Work	Present students who are struggling with a problem a different way of approaching the problem. In this case, adjust the amount of work given to a student.	Reciprocal Teaching at AdLit.org Reciprocal Teaching at Reading Rockets Reciprocal Teaching at Just Read Now!		X	
Create Multiple Versions of a Problem	Offer alternatives to a range of learners	Socratic Seminar from NBABR		X	

Adjust Pacing to Optimize Attention	Present students who are struggling with a problem a different way of approaching the problem. In this case, adjust the pacing to optimize attention.				
Promoting Student Self-Regulation and Self-Assessment					
Offer Timers to Students	As students work on tasks, they use a timer to pace their work.		X	X	X
Highlight-ing and Color-coding	Teach students to use highlighting and color-coding to identify key information.		X	X	X
Metacog-nitive Strategies	Students reflect on the problem solving process which helps them to make sense of the mathematics.	http://mathforum.org/~sarah/Discussion.Sessions/Schoenfeld.html http://www.exemplars.com/media/pdf/kathy_spruiel_meta_cognitive.pdf			
Model Problem-Solving Strategies	Teach and model strategies for: • Organization • Self-questioning • Self-monitoring • Problem solving • Memory (mnemonics)	http://crlt.indiana.edu/research/csk.html			
Rubrics	Clarify expectations for tasks by offering rubrics for scoring and rating.	http://math.about.com/od/highschool/High_School_Math_Rubrics.htm http://www.rubrics4teachers.com/mathematics.php			

Note-Taking, Summarizing, and Paraphrasing Strategies					
Double-Entry Journals (to include Cornell Notes and Two-Column Notetaking)	Note-taking strategies involving students' use of running notes complemented by identification of big ideas and important concepts—with parallel non-linguistic representations in a second or third column	Cornell Notes from JMU Word processing, concept mapping software, interactive whiteboard, document camera		X	
Messy-Notes	An informal approach to note taking (often using manipulatives such as post-it notes), with students revising the notes as they expand their understanding of text		X	X	X
Power Notes	A note-taking strategy in which students review and synthesize previous notes, highlighting and accenting major ideas, concepts, and information worth retaining	Power Notes at AdLit.org		X	X
Selective Highlight-ing	Encouraging students to highlight or underline important concepts and main ideas within a text	Selective Highlighting at AdLit.org		X	

Structured Note-taking	Purposeful and deliberate note taking in which the teacher models note-taking protocol and students use parallel structures (e.g., running notes, summaries, visual representations)	Structured Note-taking at AdLit.org		X	
Sum-It-up	Engaging students in paraphrasing and summarizing tasks at the conclusion of key juncture points within a lesson or unit	Sum-It-Up at AdLit.org		X	X
Project Organizers	Provide project organizers to help students keep track of tasks.	Word Hunts at AdLit.org	X	X	
Time Manage-ment	Provide time management cues to assist students in managing time on task.				
Writing-to-Learn Strategies That Promote Comprehension					
Open Response	Students explain their problem solving process by writing a rationale for their mathematics. Drawings are often included in this work.			X	X
RAFT	A writing assignment that articulates a clear role, audience, format, and topic	RAFT at AdLit.org Word processing, publishing software, presentation software, digital storytelling tools	X	X	X
Response Journals	Ongoing journal entries completed by students as they progress through a text or unit—often using technology-based media	Blogs, social networking sites		X	X

HYPERLINK SUGGESTIONS RELATED TO MATHEMATICS META-STRATEGIES			
Manip-ulatives	Two- and three-dimensional models and representations, encouraging students to synthesize and make connections	http://nlvm.usu.edu/	
Non-Linguistic Represen-tation	Engaging students in learning kinesthetically, visually, etc.	http://www.netc.org/focus/strategies/nonl.php	
Self- and Peer Evaluation	Students discuss and critique their own work or the work of others.	See the booklet: Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions (NCTM)	
Accountable Talk	Getting students to think speak and act like mathematicians	http://www.math.utep.edu/Faculty/duval/class/random/mathaccttalk.html	
Multiple Represen-tations	Showing mathematics in pictures, graphs, tables, and other forms generated by students	http://www.eric.ed.gov/ERICWebPortal/search/detailmini.jsp?_nfpb=true&_&ERICExtSearch_SearchValue_0=ED425937&ERICExtSearch_SearchType_0=no&accno=ED425937	
Compare and Contrast	Students compare methods of solving problems with one another and discuss the positive and negative differences of each method	http://www.ehow.com/list_5805766_compare-contrast-activities-middle-school.html	

Connections Between Concrete and Symbolic	Students learn that mathematics is the language of structure by working through and within tangible models before tying algebraic representations to them	http://www.specialconnections.ku.edu/cgi-bin/cgiwrap/specconn/main.php?cat=instruction&subsection=math/cra	
Prior Knowledge	Determining specific, previously learned skills necessary to learn a new concept and strategically organizing said skills in questions that will aid instruction.	http://www.exploratorium.edu/IFI/resources/museumeducation/priorknowledge.html	

Virginia Department of Education (VA DOE) Teacher Resources Mathematics Instruction

The following VA DOE teacher resources are listed below in detail:

- Thinking Rationally about Fractions, Decimals and Percents
- Middle School Mathematics: Online Strategies for Teachers
- 2009 Standards of Learning A.9 – Technical Assistance Document
- Thinkfinity
- Technology Sparks
- Share the Skies Telescope Project
- Probability and Statistics Professional Development Module for Elementary and Middle School Teachers (2004)
- Economics & Personal Finance Information and Resources
- Algebra I Online Tutorial
- Algebra Readiness Initiative Resources
- Algeblocks Online Training

Thinking Rationally about Fractions, Decimals and Percents
Thinking Rationally about Fractions, Decimals and Percent Instructional Activities (Grades 4-8) 2002 (PDF) – These lessons provide additional strategies for elementary and middle school teachers in the areas of fractions, decimals, percent and proportional thinking.

Activity & Page Number Related 2001 Standards of Learning Topic

- All Cracked Up 7 4.2a, 4.3 (Geometry 4.17, 5.15, 6.14) Area Model
- Animals Count 11 6.2, 7.6, 8.3, 8.17 Proportional Thinking
- Area Model of Multiplication 14 5.4, 6.6, 6.7 Area Model
- Around the World 17 4.2, 4.4, 5.1, 6.1, 6.7 Connections among Fractions, Decimals, and Percent
- Building Towers 22 6.4, 7.1 Comparing Fractions and Decimals
- Covering the Whole Unit 25 4.2 Equivalent Fractions
- Cover-up 27 4.2 Measurement Linear Model
- Decimal Multiplication 30 5.1, 5.4, 6.6 Area Model
- Dividing Fractions, Using Pattern
- Blocks 32 6.6, 7.4, 8.3 Operations with Fractions
- Egg-Carton Addition 36 4.2, 4.9, 5.7, 6.6 Operations with Fractions
- Egg-Carton Fractions 40 4.2, 4.3 Part-Whole Concept
- Four in a Row 43 4.9, 5.7, 6.6 Operations with Fractions (Addition and Subtraction)
- Fraction Strips 46 4.2, 4.3, 4.9, 6.4 Area Model
- Fraction-Strip Addition 49 4.2, 4.9, 5.7, 6.6 Operations with Fractions
- Fraction-Strip Subtraction 61 4.3, 4.9, 5.7, 6.6 Operations with Fractions
- Fraction Riddles 63 4.2, 4.9, 5.7, 6.6 Operations with Fractions (Addition and Subtraction)
- Gridlock 65 6.2, 7.6, 8.17 Proportional Thinking
- Measuring Cups 67 6.6a, 6.7, 7.4, 8.3 Operations with Fractions (Division)
- Museum Walk 70 6.1, 7.1 Connections among Fractions, Decimals, and Percent
- Paper-Folding Multiplication 71 4.2, 4.3, 6.4 Operations with Fractions (Multiplication) Area Model
- Percent Grid Patterns 75 4.4, 6.1, 7.1 Connections among Fractions, Decimals, and Percent Area Model

- Playground Problem 79 6.6a, 6.8, 7.4a, 8.3 Operations with Fractions (Addition and Subtraction) Problem Solving
- Population Density 83 6.2, 7.6, 8.3, 8.17 Proportional Thinking
- The Scoop on Ice Cream 84 7.4, 7.6, 8.3, 8.17 Proportional Thinking
- Snowy Egrets 88 6.2, 6.8, 7.6, 8.3, 8.17 Proportional Thinking
- Something's Fishy 91 4.2, 4.3 Equivalent Fractions Area/Region Model
- Spin To Win 101 4.2a, 4.3, 6.4 Comparing and Ordering Fractions
- The In-between Game 105 4.2, 4.4, 5.1, 6.4, 7.1 Decimal Relationships
- Virginia Population Density 109 6.2, 7.6, 8.3, 8.17 Proportional Thinking
- Waste Paper 112 7.6, 8.3, 8.17 Proportional Thinking
- Which Is Closer? 115 4.2, 4.3, 4.9, 5.7 Benchmarks for Fractions
- Which Is More? 120 4.3 Comparing Fractions
- Who Has 100 Things? 123 4.2, 5.2, 6.1, 7.1 Connections among Fractions, Decimals, and Percent

Middle School Mathematics: Online Strategies for Teachers
Middle School Mathematics: Online Strategies for Teachers – Virginia educators demonstrate strategies and activities that can be used to teach sixth-grade and seventh-grade mathematics.

Strategies for problem solving, vocabulary development, calculator use and building organizational skills:

- Working with Vocabulary / Concept Development (Grades 6 & 7)
  Dr. Lois Williams, VDOE mathematics specialist, on the Frayer Model for vocabulary/concept development. *Handout available:* Working with Vocabulary / Concept Development (Word)

- Multi-Step Problem Solving (Grades 6 & 7)
  Cheryl Gray, Spotsylvania County Schools middle school mathematics specialist, on how to approach multi-step problems with data presented in a table. *Handout available:* Multi-Step Problem Solving (PowerPoint)

- Scientific Calculator Use (Grades 6 - 8)
  Dr. Lois Williams explains the use of scientific calculators in Virginia middle schools. Added 10/04/07. *Handout available:* Scientific Calculator Manual (PDF)

- Vocabulary (Grades 6 & 7)
  Dr. Lois Williams shares a technique to assist students who confuse common mathematics terms such as radius and diameter.

- Notebooks for Organization (Grades 6 - 8)
  Dr. Lois Williams explains an organizational system for middle school mathematics classes. Added 10/04/07.

Computation and Estimation
Some best practices for fraction and integer concepts and operations:

- Fraction Concepts (Grades 6 & 7)
  Dr. Ena Gross, Virginia Commonwealth University (VCU) professor of mathematics education, on the prerequisites students need for fraction computation.

- Integer Models, Part I (Grade 7)
  Theresa Wills shares five different models for integer operations. Added 10/04/07.

- Fraction Computation (Grade 6)
  Dr. Ena Gross on one method sixth graders can use to approach multiple-choice fraction computation problems.

- Integer Models, Part II (Grade 7)
  Theresa Wills demonstrates the uses of five integer models for instruction. Added 10/04/07.

Number and Number Sense
Examples of properties, ratios, and order of operations teaching strategies:

- Ratios (Grade 6)
  Denika Gum, Albemarle County Public Schools sixth-grade mathematics teacher, uses manipulatives to help sixth-grade students grasp the concept of ratio.

- Properties, Part I (Grade 7)
  Noel Klimenko, mathematics specialist with Fairfax County Public Schools, describes instructional steps leading to the understanding of properties. Added 10/04/07.

- Rules for Order of Operations (Grade 7)
  Dr. Lois Williams shares an activity to get students to use writing in

mathematics to remember the rules for the order of operations. *Handout available:* Rules for Order of Operations (Word)

- Properties, Part II (Grade 7)
  Noel Klimenko demonstrates the instructional steps leading to understanding of properties. Added 10/04/07.

Geometry
Angle, similarity and polygon classification best practices:

- Angles (Grade 6)
  Dr. Margie Mason from The College of William and Mary, demonstrates strategies to develop concepts related to angles. Added 10/04/07.

- Similarity (Grade 7)
  Debi Godfrey, teacher in Henrico County, explores strategies for determining similarity. Added 10/04/07.

- Properties of Polygons (Grade 7)
  Dr. Margie Mason offers models for helping students compare and contrast classes of quadrilaterals. Added 10/04/07.

Measurement
Exploration of volume, surface area and units of measure:

- Converting Units (Grade 6)
  Dena McElligott, Virginia Middle School Mathematics Teacher Corps member in Virginia Beach Public Schools, shares a problem-solving strategy for converting units. *Handout available:* Converting Units (Word).

- Measurement in Square Units (Grade 6)
  Dr. Lois Williams offers a hands-on strategy to help students understand and convert square units. Added 10/04/07.

- Volume and Surface Area (Grade 7)
  Karen Cribbs, Hanover County Public School secondary mathematics specialist, demonstrates a hands-on introduction to volume and surface area.

- Liquid Measure (Grade 6)
  Dena McElligott demonstrates a manipulative that helps students see the relationships among the customary units of liquid measure.

- Units of Measure (Grades 6 & 7)
  Karen Cribbs models a visual approach to the concepts of linear, square and cubic units.

Statistics and Probability
Engaging activities and questioning techniques for statistics:

- Box and Whisker Plots (Grades 6 & 7)
  Brian Domroes, Spotsylvania County Schools middle school mathematics specialist, on techniques for engaging students with box and whisker plots.

- Calculators and Statistics (Grades 6 & 7)
  Eurice Dawley, of Norfolk City Public Schools demonstrates a lesson on mean and the scientific calculator. Added 10/04/07.

- Statistics (Grades 6 & 7)
  Brian Domroes provides analysis-level questions for statistics.

Patterns, Functions and Algebra
A demonstration for creating and extending patterns:

- Extending Patterns (Grades 6 & 7)
  Denise Walston, Norfolk Public Schools senior mathematics coordinator, presents an example for extending patterns. *Handouts available:* Extending Patterns Chart (Word) and Slides (PowerPoint)

VA DOE 2009 Standards of Learning A.9: Technical Assistance Document
The purpose of this Virginia Department of Education document is to assist teachers as they provide instruction and assess students on 2009 Standard of Learning (SOL) A.9. SOL A.9 is intended to extend the study of descriptive statistics beyond the measures of center studied during the middle grades. In Algebra II, students will continue the study of descriptive statistics as they analyze the normal distribution curve and normally distributed data and apply statistical values to determine probabilities associated with areas under the standard normal curve.

- Lesson 1 – Expressions and Square Roots (DOC)
- Lesson 2 – Exponents, Scientific Notation, Multiplying and Dividing Polynomials (DOC)

- Lesson 3 – Adding, Subtracting and Multiplying Polynomials (DOC)
- Lesson 4 – Factoring Polynomials, Solving Quadratics (DOC)
- Lesson 5 – Solving Equations and Inequalities, Order of Operations, Properties (DOC)
- Lesson 6 – Solving Literal Equations, Slope of Line (DOC)
- Lesson 7 – Graphing Equations of Lines, Writing Equations of Lines (DOC)
- Lesson 8 – Solving Systems of Equations (DOC)
- Lesson 9 – Relations and Functions (DOC)
- Lesson 10 – Matrices, Line of Best Fit, Measures of Central Tendency, Range, and Box-and-whisker Plots (DOC)

Thinkfinity

Thinkfinity is a Web portal to standards-based content including lesson plans, interactive programs and online resources from several national and international educational institutions and the Verizon Foundation as well as training for educators on using Thinkfinity.

Thinkfinity content providers are as follows:

- **ArtsEdge**, a program of the Kennedy Center for the Performing Arts, provides art education resources
- **EconEdLink**, developed by the Council for Economic Education, offers economic and personal finance lesson materials for K-12 teachers and students
- **EdSitement**, from the National Endowment for the Humanities and the National Trust for the Humanities, includes information in the subject areas of literature and language arts, foreign languages, art and culture, and history and social studies
- **Illuminations**, created by the National Council of Teachers of Mathematics (NCTM), provides resources for teaching math
- **Literacy Network**, a program of the National Center for Family Literacy and ProLiteracy, offers resources for literacy instruction and lifelong learning for adults and family literacy programs.
- **Read/Write/Think**, from the International Reading Association (IRA) and the National Council of Teachers of English (NCTE), includes resources for reading and language arts instruction.
- **Science NetLinks**, developed by the American Association for the Advancement of Science (AAAS), provides standards-based science resources including downloadable 60-second science updates.
- **Smithsonian's History Explorer**, created by the National Museum of American History, features resources and artifacts from the museums collections.

- **Xpeditions**, from National Geographic, offers geography resources including printable maps and an interactive atlas.

Technology Sparks
Technology Sparks, Ideas for Teachers: Integrating Technology with the Virginia Standards of Learning (PDF) resources and suggestions for incorporating technology into the Virginia Standards of Learning (SOL) middle school core curricular subjects.

Share the Skies Telescope Project
Share the Skies Telescope Project – access to research-grade telescopes allowing Virginia teachers and students to explore Australia's night skies during the school day and study astronomy without leaving the classroom.

Probability and Statistics Professional Development Module for Elementary and Middle School Teachers (2004)
Revised Probability and Statistics Professional Development Module for Elementary and Middle School Teachers(2004) – module for grades K-8 educators related to the probability and statistics strand of the mathematics standards:

- Entire Document (PDF) (DOC)
- Introduction (PDF) (DOC)
- Section 1 (PDF) (DOC)
- Section 2 (PDF) (DOC)
- Section 3 (PDF) (DOC)
- Section 4 (PDF) (DOC)
- Section 5 (PDF) (DOC)

VA DOE Economics & Personal Finance Resource
Economics & Personal Finance – information to assist in meeting the economics education and financial literacy requirements for all middle and high school students. Instruction in economics and personal finance prepares students to function effectively as consumers, savers, investors, entrepreneurs, and active citizens. Students learn how economies and markets operate and how the United States' economy is interconnected with the global economy. On a personal level, students learn that their own human capital (knowledge and skills) is their most valuable resource. In order to ensure that teachers of history/social science, career and technical education and mathematics have access to instruction in the Economics Education & Financial Literacy content areas, VDOE has produced a series of **videos for Teachers**.

Algebra I Online Tutorial
Project Graduation Algebra I Tutorial – This tool to helps to identify a student's academic needs with online pretests and provides Web-based lessons to meet those needs.

Algebra Readiness Initiative

Algebra Readiness Initiative – This site gives information and support to prepare students for success in Algebra.

Algeblocks Training

Algeblocks Training - Streaming Video – online training for teachers who either need an introduction to the use of Algeblocks or need to reacquaint themselves with the manipulative

Web-Based Teaching Resources for Mathematics

- Math.com: Math.com provides innovative ways for students, parents, teachers, and everyone to learn math. Combining educationally sound principles with proprietary technology, Math.com offers explicit directions for solving math problems from basic math through Algebra I and Calculus.

- Math Is Fun: This site provides students with multiple strategies for solving problems. There is also an option for practicing basic facts.

- Nat'l Library of Virtual Manipulatives: http://nlvm.usu.edu/en/nav/topic_t_1.html

- Mathalicious: This site offers engaging real-life problems for middle school, Algebra I, and Algebra II.

- NCTM Illuminations: This site offers online interactive lessons for kindergarten through Algebra II.

- *Gizmo Math:* Go to www.explorelearning.com and browse for Gizmo math simulations by state correlation.

- Visual Fractions: A tutorial that models fractions with number lines or circles.

Professional Mathematics Resources for Teachers

- National Council of Teachers of Mathematics (NCTM)
  http://www.nctm.org/about/default.aspx?id=166
  The National Council of Teachers of Mathematics is a public voice of mathematics education supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development and research.

- National Council of Supervisors of Mathematics (NCSM)
  http://www.mathedleadership.org/about/index.html

- National Math Panel Report
  http://www2.ed.gov/about/bdscomm/list/mathpanel/index.html
  On March 13, 2008, the National Mathematics Advisory Panel presented Foundations for Success: The Final Report of the National Mathematics Advisory Panel to the President of the United States and the Secretary of Education. In response to a Panel recommendation, the U.S. Department of Education, in partnership with the Conference Board of Mathematical Sciences, hosted the first National Math Panel Forum on October 6-7, 2008. The Forum brought together various organizations and other interested parties to use the Panel's findings and recommendations as a platform for action.

- TIMSS (http://nces.ed.gov/timss/)
  The Trends in International Mathematics and Science Study (TIMSS) provides reliable and timely data on the mathematics and science achievement of U.S. 4th- and 8th-grade students compared to that of students in other countries. TIMSS data have been collected in 1995, 1999, 2003, and 2007.

- U.S. Participation in International Assessments
  http://nces.ed.gov/surveys/international/pdf/brochure_USparticipation.pdf
  Learn about PIAAC, PIRLS, PISA, and TIMSS, how they compare with one another, and which countries participate.

- APEC Education Network
  http://hrd.apecwiki.org/index.php/Education_Network_%28EDNET%29
  The goal of the APEC's education activities is to foster strong and vibrant learning systems across APEC member economies, promote education for all, and strengthen the role of education in promoting social, individual, economic and sustainable development.

- Inside Mathematics: www.insidemathematics.org

  Inside Mathematics is a professional resource for educators passionate about improving students' mathematics learning and performance.

This site features classroom examples of innovative teaching methods and insights into student learning, tools for mathematics instruction that teachers can use immediately, and video tours of the ideas and materials on the site.

Resources for Parents

- A Family's Guide: Fostering your child's success in school mathematics, National Council of Teachers of Mathematics, 2004. www.nctm.org

- Beyond Facts and Flashcards: Exploring Math with Your Child by Jan Mokros, Heinemann, 1996.
  http://www.heinemann.com/products/08375.aspx

- Virginia Department of Education Parent Links:
  http://www.doe.virginia.gov/students_parents/index.shtml
  This link gives quick and easy access to many areas of interests for parents.

- NCTM Family Resources:
  http://www.nctm.org/resources/families.aspx

- Figure This – Math Challenges and Activities for Families
  http://www.figurethis.org/fc/family_corner.htm

- *Everyday Mathematics* Parent Connections
  https://www.wrightgroup.com/parent_connection/index.html
  Tips and activities for each grade level

- Math Forum
  http://mathforum.org/
  The Math Forum is the leading online resource for improving math learning, teaching, and communication since 1992. We are teachers, mathematicians, researchers, students, and parents using the power of the Web to learn math and improve math education.

Making Mathematics Accessible for English Language Learners

Introduction

In order to help all children develop a comprehensive understanding of mathematics, teachers must possess depth of mathematical content knowledge as well as a repertoire of pedagogical skills and strategies with which to teach. They must also know their students well as learners in order to match instruction to student needs. While the content students are to learn remains constant for all students of a specific grade level or course, the pedagogical strategies used may vary according to the specific needs of the learners. For English Language Learners (ELLs), language acquisition is a specific need that also must be addressed as children learn mathematics. The following sections specify how to link language acquisition to the content and pedagogy of mathematics through careful assessment and planning.

Linking Language Acquisition to Mathematics

Alexandria City Public Schools use the WIDA levels of English language proficiency to identify a student's level of English language acquisition. Developed by the World-Class Instructional Design and Assessment Consortium, WIDA levels are used to determine the type and amount of service provided to ELLs. There are six levels ranging from level 1 – entering to level 6 – reaching, with each level indicating how students understand, process, produce or use language (definitions of the WIDA levels can be found at http://www.wida.us/standards/PerfDefs.pdf). WIDA levels are useful to classroom teachers because they help teachers determine how to scaffold instruction for ELLs in each of the four English domains – listening, speaking, reading, and writing.

Mathematics has its own specific language that is central to understanding the content. In order for English Language Learners to communicate their mathematical understandings, they need to acquire both the conversational (social language) and the academic language associated with mathematics. Therefore, it is imperative for teachers of ELL's to embed mathematical vocabulary and language structures into their daily lessons, without taking the focus of the lesson away from the math itself.

What Students Can Do

The following chart, developed by the WIDA Consortium, provides descriptions of what ELLs can do with support.

The WIDA CAN DO Descriptors work in conjunction with the WIDA Performance Definitions of the English language proficiency standards (http://www.wida.us/standards/PerfDefs.pdf). The Performance Definitions use three criteria – linguistic complexity, vocabulary usage, and language control – to describe the increasing quality and quantity of students' language processing and use across the levels of language proficiency.

What Teachers Must Do

English Language Learners have very unique linguistic needs that may prevent them from learning and communicating mathematics effectively. Therefore, it is the teacher's responsibility to use a variety of techniques to make the language and content more accessible when presenting and communicating information. In *Making Content Comprehensible for English Learners: The SIOP Model* (2008), Echevarria, Vogt, & Short recommend three specific techniques to make lessons more accessible.

1. Clarity of speech
   - Speak slower.
   - Use shorter sentences.
   - Avoid idioms and nonsense words.
   - Match language modifications need to students' language proficiency levels.

2. Clear explanation of tasks
   - Present directions in a step by step manner.
   - Demonstrate or provide a model of what students are expected to do.
   - Include written directions with all oral directions.

3. Variety of techniques
   - Consciously use body and facial gestures to facilitate communication.
   - Give silent "think time" after all questions and before students interact with one another to allow all students to formulate their own mathematical thoughts and English learners to think about how to express their ideas in English.
   - Make lessons as visible as possible through the use of posters, notes, graphic organizers, and equations.

Assessing

Because English Language Learners are so diverse, there is no single mathematics assessment that will provide all the information necessary to best serve the students. Therefore, a variety of assessments should be used to determine students' mathematical knowledge. For this reason, the following types of assessment should be considered before making judgments of where students are along the learning continuum.

Diagnostic Assessments (Pre-Assessments)

When planning for ELLs, it is important to have the most accurate information possible about what students know and understand. The purpose of diagnostic assessments is to gather information about what students know in order to differentiate teaching and learning. They may be given at the beginning of a year, before a unit, and when new students arrive in the classroom from another country or school. Diagnostic assessments may be used to:

- To provide baseline data about students' knowledge
- To determine students' prior knowledge

- To identify areas of strength
- To identify misconceptions and gaps in learning

While there are some formal diagnostic assessments such as end of year tests from the previous grade level and pre-tests for new units, informal diagnostic assessments often yield valuable information about students' mathematical knowledge. Informal assessments may include interviews, simple tasks, and games. An added benefit to informal assessments are that they may be less stressful for English Language Learners.

Formative Assessments

Formative assessments provide immediate, relevant feedback to both teachers and students. The evidence collected is not typically graded, but rather used by teachers to adjust instruction, and by students to clarify their learning. Because of the language barriers many ELLs experience in the classroom, they may be less confident in their mathematical abilities. By providing immediate feedback, teachers build ELLs' self confidence and address misconceptions before they become entrenched in the students' minds. There are numerous examples of formative assessments such as observation, questioning, student response boards, exit slips, interviews, thumbs up, and checklists.

Summative Assessments

Summative assessments are designed to evaluate the extent to which students have achieved the learning goals of a unit, course, or year. For ELLs, summative assessments may need to be adjusted to make the language more accessible without taking away the complexity of the math. Tests, quizzes, academic prompts, and projects are examples of summative assessments. The following adjustments provide support for ELLs with varying levels of language proficiency, and may be used to create and administer assessments:

- Simplify directions
- Simplify the language of the test
- Provide word banks
- Include illustrations
- Read the test aloud
- Allow extra time for test completion
- Allow students to respond orally rather than in writing when appropriate

Integrating Pedagogical Strategies

The strategies presented in this section are research-based, and specifically known to increase the level of mathematical understanding for English language learners (Coggins, 2007; Echevarria, 2008; Hill, 2006; WIDA Consortium, 2007). They provide strategic support that can be used throughout lessons to help students focus on the math content as they develop proficiency in the target language.

Non-Linguistic Representations

Nonlinguistic representations are ways teachers communicate knowledge to students to help them recall, think, and understand information. These nonlinguistic representations allow students to create mental pictures related to the concept. Some examples are:

- Real objects and visuals such as photographs, graphs and charts
- Demonstrations that use actions to show and convey meaning
- Pointing, gestures and movement to give directions or explain
- Hands-on activities to help students connect and build concepts
- Physical and technological models
- Manipulatives

Graphic Organizers

Graphic organizers (e.g. Venn diagrams, concept maps, webs) are visual charts or diagrams that provide an organizing structure for a collection of ideas about a concept or topic. They help ELLs understand and communicate ideas and processes. They can be used at various points throughout a lesson or unit, depending upon the purpose. Though they may be created by teachers with students, they are most helpful when students create their own. Additional information about graphic organizers can be found in Chapter 5 of *English Language Learners in the Mathematics Classroom* (Coggins, et al., 2007).

Advance Organizers

Advance organizers are organizational frameworks created by the teacher and include the most important information about the math concept in a unit. They are provided at the beginning of a unit to help ELLs access prior knowledge and understand what they are expected to learn. Advance organizers can be nonlinguistic, linguistic or a combination of both depending upon the age and level of the ELLs. In *Classroom Instruction That Works with English Language Learners*, Hill & Flynn (2006) identify four types of advance organizers:

- Narratives – used to tell a story

- Expository – for describing new content
- Skimming – to use before reading texts (e.g. reference books, textbooks)
- Graphic – a visual representation of new content

Sentence Frames

Sentence frames help students develop the oral language of mathematics they need to fully participate in math discussions. They help students use the vocabulary in meaningful context and allow them to complete sentences and thoughts that are grammatically correct. After sufficient practice with using the frames to express their mathematical thinking, students will be ready to use the frames for writing.

For example, the following frames support students at various language levels in their discussions about polygons.

Beginning Level
This is not a polygon. It has curves.

Intermediate Level
This is not a polygon because it has curves, and is open.

Advanced Level
This shape has four straight sides, four vertices, and is closed; therefore, it is a polygon.

Prompts to Support Student Responses

Prompts can help English language learners get started when responding to a question. When you encourage them toward an answer, they are more likely to follow your lead and respond with confidence. For example, begin with: "You figured it out by..." or "It is a polygon because..." or "First solve this equation, then....".

Discourse

The exchange of ideas through discourse provides opportunities for students to ask questions, clarify their thinking, and learn from one another. By listening to students explain, teachers gain entry into students' assumptions, misconceptions, and understandings. This type of formative assessment helps teachers determine what a student needs next in order to grow.

Mathematical discourse involves both dialogue, when ideas are shared without judgment, and discussion, when trying to reach a decision or justify an answer. Dialogue would be appropriate for sharing multiple strategies about an operation or problem solving task. Discussion would be appropriate when determining which answer is correct and why. And either might be appropriate for helping students make generalizations about the big ideas of mathematics.

One way to make discourse more accessible for ELLs is through rehearsal – when students practice privately what they will say prior to speaking publicly. Another strategy is to begin with a conversation in which they know a lot before asking them to share new or recently learned information. Wait time is also essential because it allows time to think and generate a response.

Accountable Talk Moves

In order for students to participate fully in mathematical discourse, they need to be able to explain their own ideas as well as listen and understand their peers' ideas. Discourse, like any other skill, needs to be taught and practiced in order for students to use it with ease and comfort. The following talk moves help teachers and students become more skilled in the use of discourse.

- **Turn and Talk –** Each student turns to a partner and discusses the question posed by the teacher. With ELLs who are not confident speaking in English, you might pair them with a partner who answers the question and have the ELL repeat the partner's words. Another option is to give the ELL the question before the lesson begins and help him or her decide what to say. If there are students who speak the same native language, then you may allow them to talk first in their native language and then in English.

- **Reason or Justify – How do you know?** Asking students to justify their answers and explain their strategies holds them accountable for learning and helps them practice formulating and expressing their thoughts. Teachers also get a better picture of what students really understand about a concept. This information may not always be visible in students' written work.

- **Restate – Can you repeat what ________ just said?** Asking students to restate what another person has said helps them improve their listening skills. For ELLs it has the added benefit of giving them the opportunity to hear the idea a second time. Additionally, students realize their ideas are valued and being heard by others.

- **Revoice –** Paraphrasing what a student says may be used to clarify what a student has said, emphasize a really important concept the teacher wants students to discuss, or connect students' comments to the mathematical goal.

- **Prompting – Can anyone add to what _____ said? Tell me more.** Prompting is used to involve more students in the conversation and to allow for multiple ideas to be discussed.

- **Wait time –** Waiting for at least five seconds after asking a question gives ELLs time to process the question and formulate a response. Often, ELLs think about concepts in their native language and need time to translate their thoughts into the target language.

Questions

Questions should vary by level of difficulty. It is important to expose all students to lower level comprehension and application questions as well as higher level synthesis and evaluation questions. Carefully consider the students' fluency with the language and the math concept before deciding who will respond. In *English Language Learners in the Mathematics Classroom*, Coggins, the authors (Coggins et al., 2007) note that although questions and responses asked of English language learners may vary according to their language proficiency levels, even level 1 students are capable of responding to higher level questions and should be included in all classroom activities. It is important to move students along as they become more proficient with the English language. Examples of questions, supports, and student responses for the various language proficiency levels are presented in the following table:

WIDA Level	Questions	Supports	Student Responses
Level 1 – Entering	- Yes/ no questions *Is this a triangle?* - questions with choices *Point to the inequalities.*	-Pictures or gestures along with the question -Sentence frames -Include higher level questions	One word responses or phrases *Yes.*
Level 2 – Beginning	- Five W's (Who, What, Where, When, Why, and How) *What do you call this measuring tool?*	- Pictures or gestures along with the question - Sentence frames -Include higher level questions	Short phrases and simple sentences *This is a ruler.*
Level 3 – Developing	- Questions that require more than a one sentence answer *What are three attributes of rectangular prisms?*	- Pictures or gestures along with the question - Academic language posted and visible -Include higher level questions	Expanded sentences with some errors *It don't got curves. There is eight vertices. There is six faces.*
Level 4 – Expanding	Ask more how and why questions to elicit more language and academic vocabulary *How are fractions and decimals similar and different?*	- Pictures or gestures along with the question - Academic language posted and visible	Variety of sentence lengths and levels of complexity *They name the same amount. They have the same value. Fractions show the total number of equal parts in the denominator, but decimals don't.*
Level 5 – Bridging	Continue to ask higher level questions, just as you would of native English speakers		Approaches the language level of their school-age peers
Level 6 – Reaching	Ask the same questions of ELLs as school-age peers		Comparable to native English speakers

Adapted from:

Coggins, D., Kravin, D., Coates, G., & Carroll, M. (2007). *English language learners in the mathematics classroom* (pp. 84–86). Thousand Oaks, CA: Corwin Press. &

WIDA Consortium. (2007). *English language proficiency standards and resource guide: Prekindergarten through Grade 12*. WI: Board of Regents of the University of Wisconsin System.

Multiple Representations

Multiple representations in mathematics are different ways students can represent ideas, abstract concepts or relationships. These mathematical representations help students develop their communication, reasoning and problem-solving abilities; make connections among ideas; and learn new concepts and procedures. Some examples are:

- Number models
- Graphs
- Pictures
- Manipulatives
- Diagrams or tables

Compare & Contrast

Compare and contrast strategies require students to distinguish similarities and differences among items or ideas. By comparing and contrasting, ELLs are able to learn mathematics at a deeper level because they are forced to activate prior knowledge, make connections, and draw conclusions. Opportunities to compare and contrast materialize in every mathematical strand – in algebra when students analyze patterns; in operations when students compare answers to estimates; and in geometry when analyzing attributes of shapes – and should become an integrated part of students' daily routines.

Vocabulary Charts and Word Walls

Vocabulary charts and word walls are tools teachers can use to help students acquire and increase their academic vocabulary as they develop their mathematical knowledge. Charts and word walls should include the words along with a nonlinguistic representation, such as:

- A word with a picture
- A word with an example
- A word with the real object
- A group of words related a concept
- A word with words that share a common root "cognates"

A word wall only includes relevant content vocabulary related to a concept or a unit. These words are used throughout the lesson, and students are encouraged to use them in their writing and discussions. They should be listed alphabetically.

Matching Informal Language to New Math Vocabulary and Concepts

ELL students frequently use informal language to describe their mathematical ideas. Teachers should help ELLs match this informal language to the specific mathematic vocabulary. An example of this could be how the word *integer* refers to positive and negative numbers. With Beginning ELLs, teachers could relate integers to having money and owing money (money I have to pay back).

Metacognition

In essence, metacognition means thinking about one's thinking. It is the process of reflecting upon one's work to make sense of what one understands and needs to learn. Having students reflect upon strategies they used or how they solved a particular problem teaches ELLs how to self-monitor their own learning and ask questions when they do not understand something.

Games

Mathematics games offer ELL students a non-threatening environment to practice and develop fluency in their skills. It also provides them opportunities to have frequent meaningful encounters with mathematics vocabulary. Teachers can support students by:

- Group students in pairs or small groups
- Group students who are in different language competence and math levels
- Pair students with someone who speaks the same native language (While it is helpful for students to be able to speak their native language, it is also important to pair them with students who encourage the use of the target language and know how to guide rather than tell.)

Planning Lessons for English Language Learners

Careful planning is essential in order to implement meaningful lessons for English Language Learners.

Before the Lesson…

Consider…	Ask yourself…	Select Appropriate Strategies…
Big ideas, essential questions, and mastery objectives	*What is the specific learning target?*	Refer to curriculum maps and student assessment data

Before the Lesson…

Consider…	Ask yourself…	Select Appropriate Strategies…
	What will students do to demonstrate they have achieved this target?	
The type of language – reading, writing, speaking, listening – students will need in order to demonstrate understanding	*What kind of language are students expected to reproduce – reading, writing, and speaking, listening?* *How will students demonstrate their understanding of the mastery objective (e.g. discuss, write, explain, describe, read).*	Match informal language to new math vocabulary and concepts Multiple Representations Compare and Contrast Sentence Frames Discourse
Possible language barriers	*What strategies can be woven into the lesson to eliminate language barriers?*	Nonlinguistic Representations Sentence Frames Advance Organizers
Launching the lesson	*How will I hook and engage student interest?* *How do I access students' prior knowledge?*	Advance Organizers Nonlinguistic Representations Questions Role Plays

During the Lesson …

Consider…	Ask yourself…	Select Appropriate Strategies…
Learning activities and sequence	*How will students know where they are going and how they will get there?* *In what parts of my lesson will I incorporate the*	Graphic Organizers Vocabulary Charts and Word Walls Advance Organizer

<u>During the Lesson …</u>

Consider…	**Ask yourself…**	**Select Appropriate Strategies…**
	essential question(s)? *How will I address key academic vocabulary throughout the lesson?* *How will students be asked to revisit, revise, rethink, and refine their learning?*	Formative Assessment Metacognition Games
Scaffolding	*What kind of supports will my ELLs need?* *How will I group students?*	Nonlinguistic Representations Prompts Discourse Turn and Talk Questions
Formative Assessments	*What <u>formative assessment tasks</u> will students complete to help me give them feedback so that they can adjust their learning process? (e.g.,)*	Higher-order Questions, Turn and Talk Justify Restate Observations
Lesson closure	*How can I have students summarize what they learned?*	Prompts Sentence Frames
Homework	*How will I use homework to help students extend and refine their learning?* *How will homework be differentiated so kids can access it?*	Games Simplify Directions Nonlinguistic Representations

References

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Strategies for Differentiation for Special Education

1. Emphasize Scaffolding:

- Break lessons or tasks into segments that are smaller and easier for students to focus on.
- Organize the lesson segments incrementally, helping every learner move from initial acquisition toward growing levels of independent modeling and guided transfer.

2. Differentiate Content, Process, and Product:

- Maintain common standards for all students (Stage One).
- Differentiate content, allowing for differing levels of complexity.
- Use tiered reading assignments to concentrate upon parallel content but accommodating differing reading levels.
- Incorporate tasks that are aligned with specific student interests.
- Analyze and address students' learning profiles (e.g., sensory modalities, introversion/extraversion, comfort levels with varying grouping configurations).
- Differentiate process, adjusting whole group, small group, and one-on-one structural arrangements within the classroom.
- Adjust level of difficulty of tasks.
- Break complex tasks into smaller parts.
- Adjust amount of time for tasks.
- Adjust amount of required work.
- Create multiple versions of a problem in order to offer a range of alternatives for learners.
- Adjust pacing to optimize attention (e.g., the 10/2 rule—For every ten minutes of teacher-led instruction, allow for at least two or more minutes of student interaction and processing).

3. Encourage students to have multi-sensory experiences:

- Address students' learning modality preferences.
- Engage students in sensory explorations involving key mathematical concepts and processes.
- Allow for appropriate movement and student interaction with their learning environment.

4. Take a section of an assignment (e.g., a worksheet) and enlarge it:

- Visually organize information so that students "bracket" key sections.
- Emphasize a specific section visually, helping students to focus and concentrate.

5. **Revisit guided instruction to ensure that students internalize key mathematical concepts and processes:**

    - Start with some form of hook or anticipatory set activity to frame learning.
    - Establish student understanding of lesson objectives.
    - Model and encourage student replication of modeled behaviors.
    - Move toward some level of guided practice.
    - If time permits, encourage students to move toward some level of independent practice.
    - Bring closure to the lesson, ensuring that students have attained understanding of its purpose and main ideas.

6. **Use graphic representations and other visual organizers to reinforce students' concept development:**

    - Use advance organizers such as outlines, bullet points, and outlines to help students track the flow or organization of a lesson or task.
    - Use graph paper to help students guide their calculations so that they are aligned with proper place value.

7. **Use a Multi-Step Approach to Academic Vocabulary Instruction:**

    - Present and model examples, illustrations, and descriptions of key terms.
    - Quickly have students model and replicate these ideas.
    - Have students create non-linguistic representations of key terms.
    - Ask students to discuss terms with one another.
    - Use periodic extending and refining activities to ensure that students understand key terms and related academic vocabulary phrases.
    - Involve students in games and friendly competitions using key academic terms.

8. **Use Previewing and Critical-Input Experiences:**

    - When introducing new content, make certain that students preview it to make connections with prior learning.
    - As new content is introduced, engage students in short and experience-based explorations to help them activate or build new cognitive schema.

9. **Align Students' Work with Mathematics and Reading:**

    - Emphasize core academic vocabulary before assigning problems.
    - Revisit and encourage students to apply key vocabulary before applying it.
    - Have students create visual representation of key concepts and terminology.

10. Framing the Lesson:

- Introduce the Essential Question(s)/objectives(s)-Make the language accessible for all students (words, pictures, phrases, to promote clarity).
- Activate Prior Knowledge- Discover and acknowledge what students already know as it relates to the essential questions. For example, by making connections to their everyday life.
- Establish the criteria for success.

11. Making Connections Between Math Topics:

- Assist students in making connections to previous taught lessons and topics.

12. Questioning/Wait Time:

- Being able to scaffold questions to develop student understanding. As questions are asked, provide wait time, it allows for students to process the question and a response for it. It also allows for misconceptions to be clarified. Build a routine that promotes wait time, for example; Think, Write, Share.

13. Classroom Discourse:

- "Research from around the world validates the importance of dialogue as a key avenue for learning content with understanding and developing reasoning, social skills, and intelligence" (Douglas Reeves, Richard Allington, Vgotsky). Discourse strategies include; Revoicing, Students restating someone else's reasoning, Apply one's reasoning to someone else's reasoning, adding on to a student's response, using wait time (Chapin, O'Connor, Anderson).

14. Concrete Modeling:

- A variety of materials/manipulatives should be available for all students, all the time. For example; pattern blocks, counters, base-10 blocks, slates, and markers.

15. Visual Models:

- This allows students to connect the language in the classroom to pictures. Examples include; number lines, frames and arrows diagrams, part/part/total diagrams.

16. Modeling Physically:

- Using physical gestures to reflect the content being taught.

17. Providing Organizational Tools:

- This includes; part/part/total, Venn Diagrams, place value charts.

18. Using Technology:

- Allowing students to have access to resources such as; Alpha Smart, calculators, recording devices, interactive websites to promote further understanding.

19. Break Tasks into Smaller Parts:

- Present students who are struggling with a problem a different way of approaching the problem. In this case, provide students with the problem broken down into smaller parts.

20. Use Friendlier Numbers:

- Present students who are struggling with a problem a different way of approaching the problem. In this case, provide students with friendlier numbers (smaller numbers) to solve word problems.

21. Have Students Chart Their Own Progress:

- Introduce rubrics or scoring guides (such as checklists) to help students understand the evaluation criteria for which they are responsible.
- Ensure that every measurement topic is aligned with specific data/feedback that students can record in their academic notebooks.
- Ask students to monitor their own progress by examining data patterns and trends (e.g., To what extent am I demonstrating progress in relationship to each measurement topic?)

22. Building Students' Independence:

- Offer timers to help students with pacing.
- Teach highlighting and color coding of key ideas and academic terms.
- Use think-alouds and other metacognitive strategies.
- Teach and model strategies for: (a) organization, (b) self-question and self-monitoring, (c) problem solving, and (d) memory tools (e.g., mnemonics).

23. Summarizing/Closing the Lesson:

- Exit slips, reflective journals allow students to reflect on the concepts and skills covered. It also allows for them to elaborate on any misconception.