ALGEBRA I
# Algebra I

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## Algebra 1 - Unit One  
### Quarter 1: 2 Weeks

### What Comes Next? Representing Numerical Patterns

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### Stage One Desired Results

**Transfer Goals: Students will...**

Construct the relationship among patterns, models, word problems, tables, and graphs through active learning experiences.

### Enabling Knowledge Objectives (Know/Do):

#### Declarative Knowledge: Students will know...

1. Vocabulary terms: patterns, tables, graphs, symbols, rule, expression, relationship
2. Patterns can be represented in many ways and described using words, tables, graphs, and symbols.
3. Mathematical relationships exist in patterns.
4. A rule, an expression, can be generated from a pattern.

#### Procedural Knowledge: Students will be able to...

1. Describe the relationship found in numerical patterns, using words, tables, graphs, or a mathematical sentence and symbols to express the relationship.

### Essential Questions

1. How can we find and analyze the relationship among numerical patterns?
2. How can we represent numerical patterns to model real-world situations?

### Virginia Standards of Learning

**Virginia Standards of Learning addressed in this unit: A.1**

### Suggested Resources:

1. **NCTM Navigating Through Algebra**
   - Bouncing Tennis Balls (pp. 21-24)
   - Walking Strides (pp. 25-26)
2. **NCTM Illuminations**
3. **Vocabulary**
   - Virginia Department of Education Grades Algebra I
4. **Gizmo Math: Pattern Activities**
5. **Mathalicious**
6. **VA DOE Algebra I Curriculum Framework**
7. **VA Enhanced Scope and Sequence:**
   - Traffic Jam p. 18
**Stage Two: Assessment Evidence**

**Diagnosis/Pre-Assessment:**

Students will complete 3-6 open response problems based on misconceptions dealing with representing and analyzing patterns. The data from this pre-assessment will be used to plan and differentiate for the instruction. Additionally, diagnosis/pre-assessment should determine students’ readiness levels and background knowledge related to: (1) interpreting patterns, (2) representing patterns, and (3) the generalization of a rule or expression.

**Culminating/ Anchor Performance Assessment Task(s)**

(Designed to evaluate the extent to which students have achieved transfer/understanding at an independent level relative to key mastery objectives and major unit understandings/essential questions):

**Get Ready for a Marathon**

You are the personal trainer for a client who is going to run a 26 mile marathon just 32 weeks from today!! Right now he/she can run 10 miles, but you need to increase his/her stamina and endurance to be able to run the full marathon. Create a running routine that will increase his/her distance by a constant amount every week to get you him/her ready in time for the race. You will need to present and discuss this plan with your client whom will be represented by your teacher and the class.

The resulting plan should include an analysis of how you determined your plan and reasoning for your decisions – be sure to include your table and graph! (NOTE to Teacher: Prepare 5-6 client profiles in advance to assign to various students. Example: Client 1 is 35 year old female who has never run a marathon but is in decent shape and can run 5 miles a week. Client 2 is a 22 year old male that is in peak shape and can easily run 12 miles a week.)
Rubric for Culminating/Anchor Performance Assessment Task:

4 = Student demonstrates a high degree of transfer and independent application involving mathematical problem solving, reasoning, communication, representation, and making connections in response to authentic problems.

3 = Student demonstrates an ability to engage in guided transfer involving mathematical problem solving, reasoning, communication, representation, and making connections in response to authentic problems.

2 = Student demonstrates essential knowledge of representing and predicting patterns, but makes mistakes or shows misunderstandings when engaged in independent problem solving and reasoning.

1 = Student demonstrates confusion or difficulty dealing with both basic and independent aspects of problem solving involving representing and predicting patterns.

Additional Performance Tasks and Academic Prompts:

(Spiraling in Their Design to Lead towards Student Understanding and Growing Levels of Transfer): Prior to working on patterning activities in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Real-life word problems involving patterning, the prediction of what comes next using recursion, and the development of a rule
- Models of textbook problems that make use of a range of modalities, including visual representations and illustrations.
- Explain patterns using illustrations, numbers, diagrams, and number lines.
- Academic Prompt: Given a dot pattern, create a real-world situation or scenario to illustrate the pattern.

Other Evidence (quizzes, tests, observations, dialogues, work samples):

1. Tests and Quizzes: Students should complete a minimum of three tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., patterns, tables, graphs, symbols, rule, expression, relationship)

2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to patterning.
Student Self-Assessment and Reflection:

1. Exit Slips: At the conclusion of class, students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about analyzing pattern problems? Are there areas in this unit that you still find difficult to understand? How would you explain a key concept from today’s lesson?

2. Self-Reflection and Self-Assessment Tasks: Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.

3. Accountable Talk: Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (How do we use patterns in our everyday lives?) to hook students into considering the effects of math on their lives.

2. Introduce the essential questions and discuss the culminating unit performance tasks (working as a personal trainer).

3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms. (A possible vocabulary model is the Frayer Model.)

4. Present concept lessons on representing patterns using different shapes and real world situations. Then, have students compare the results for their representations.

5. Introduce pattern block or tile growing patterns. Students work in groups to develop efficient strategies for predicting the 10th, 50th, and 100th term.

6. Use VA Enhanced Scope and Sequence fraction lesson, “Square Patio”, to engage students in real life pattern scenarios.

7. Present concept lessons that connect patterns to tables, graphs, and rules.

Suggestions for Scaffolding

1. Maintain a word wall and notebook log of key vocabulary words along with pictures and kid friendly definitions.

2. Use a think-aloud format to demonstrate the steps in predicting subsequent values in patterns.

3. Use tiles or pattern blocks as concrete models to represent the pattern sequence. Then assist students in seeing the connection to the table, graph, and symbolic representation.

4. Illustrate the patterning sequence using a table and graph.
5. Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.

6. Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative options for students who can make alternative choices about approach and format.
Express Yourself! Using Expressions, Solving Linear Equations & Inequalities

Stage One: Desired Results

Transfer Goals: Students will...

1. Represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.
2. Solve multi-step linear equations and inequalities with one variable.

Enabling Knowledge Objectives (Know/Do):

Declarative Knowledge: Students will know...

1. Key vocabulary terms: expression, equation, inequality, verbal expression, variable, literal equation
2. Mathematical modeling involves creating algebraic representations of quantitative real-world situations.
3. The numerical value of an expression is dependent upon the values of the replacement set for the variables.
4. There are a variety of ways and algebraic conventions used to compute the value of a numerical expression and evaluate an algebraic expression.
5. The operations and the magnitude of the numbers in an expression impact the choice of an appropriate computational technique.
6. A solution to an equation or inequality is the value or set of values that can be substituted to make the equation or inequality true.
7. Real-world problems can be interpreted, represented, and solved using linear equations and inequalities.
8. The process of solving linear equations can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations.

Essential Questions

1. How can expressions, equations, and inequalities help us to generalize and describe patterns in our world?
2. How can we use expressions, equations, and inequalities to model and solve real-world problems?

Virginia Standards of Learning (SOLs) in this unit: A.1, A.4, A.5

Solving Two-Step Equations

\[
\begin{align*}
2x + 3 - 3 & = 7 - 3 \\
2x & = 4 \\
\frac{2x}{2} & = \frac{4}{2} \\
x & = 2
\end{align*}
\]

Simplify.

Divide each side by 2.

Simplify.
9. Properties of real numbers and properties of equality can be used to justify equation solutions and expression simplification.
10. Properties of inequality and order can be used to solve inequalities.
11. Set builder notation may be used to represent solution sets of equations.

Procedural Knowledge: Students will be able to...

1. Translate verbal quantitative situations into algebraic expressions and vice versa.
2. Model real-world situations with algebraic expressions in a variety of representations (concrete, pictorial, symbolic, verbal).
3. Evaluate algebraic expressions for a given replacement set to include rational numbers.
4. Solve a literal equation (formula) for a specified variable.
5. Simplify expressions and solve equations, using the field properties of the real numbers and properties of equality to justify simplification and solution.
6. Solve multistep linear equations in one variable.
7. Confirm algebraic solutions of linear equations, using a graphing calculator.
8. Solve and graph multistep linear inequalities in one variable.
9. Justify steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers.
10. Solve real-world problems involving inequalities.
Suggested Resources:

1. **Zaccaro:**
   - Chapter 2 Solving Equations
   - Chapter 3 Using Algebra to Solve Equations

2. **Prentice Hall, Algebra I:**
   - Extension: Developing Geometric Formulas p. 116
   - Investigation: Modeling Equations p. 95
   - Technology: Graphing to Solve Equations p. 102
   - Extension: Interpreting Solutions p. 160
   - Algebraic Reasoning p. 173

3. **Virginia Department of Education Resources**
   - [Algebra I Standards of Learning – 2009 Version](#)
   - [Algebra I Curriculum Framework](#)
   - [Algebra I Enhanced Scope and Sequence](#)
     - Cover-up Problems
     - Algebblocks and Equation Solving
     - Greetings
     - Solving for y
     - Solving Linear Equations
     - A Mystery to Solve
     - Solving Inequalities
   - [Mathematics Crosswalk Between the 2009 and 2001 Standards of Learning](#)
   - [Mathematics Vocabulary](#)

4. [Mathalicious](#)
5. [NCTM Illuminations](#)
6. [NCTM Navigating Through Algebra](#)
7. **Gizmo Math:** Go to [www.explorelearning.com](http://www.explorelearning.com) and browse for Gizmo by state correlation. Some examples are:

   - **Order of Operations**
   - **Using Algebraic Expressions**
Modeling and Solving Two-Step Equations

Solving Two-Step Equations

Solving Inequalities Using Multiplication and Division

Modeling One-Step Equations - Activity A
Modeling One-Step Equations - Activity B

Using Algebraic Equations

Solving Linear Inequalities using Addition and Subtraction
Solving Linear Inequalities using Multiplication and Division
Express Yourself! Using Expressions, Solving Linear Equations & Inequalities

Stage Two: Assessment Evidence

**Diagnosis/Pre-Assessment:**

Students will complete 8-10 open response problems dealing with solving equations and inequalities, order of operations, substitution, and writing expressions. The data from this pre-assessment will be used to plan and differentiate for the instruction.

**Culminating/Anchor Performance Assessment Task(s)**

**City Planner Proposal for Potomac Yard**

This project will evaluate student’s ability to construct relationships between patterns, models, word problems, tables and graphs in a real world situation. Students will model real world situations with algebraic expressions in a variety of representations (concrete, pictorial, symbolic and verbal).

Potomac Yard is being renovated to allow for a new Metro Station, Condos, Town Houses, and Stores. The length of the space is 1600 feet and the width is 1600 feet. The border is going to be “Green” with plants, trees, gardens, bike paths and walkways. We need to figure out the largest width that can be set aside inside the perimeter for the “Green” border. The area of Potomac Yard is 12,000,000 square feet. The area for the renovation must be at least 9,000,000 square feet of retail, parking and living space, but no more than 10,500,000 square feet.

**Academic Prompt:**

1. Make predictions for the width of the “Green” border.
2. Calculate the area of the rectangle inside the border for the renovation based on each chosen width.
3. Make a table with the area of the renovation as the y value and the width as the x value.
4. Graph each ordered pair.
5. Discuss the possible options for the renovation based on the data.
Assessment:

1. Decide which option works best for the “Green” border.

2. Explain in writing why you picked that particular width. For example, more “Green” for the environmentalist or more retail space for the merchants.

3. Produce a pictorial representation of your final decision on a poster or power point. Make it to scale using the proper measurements and include a key representing plants, trees, gardens, walkways, buildings, parking lots, etc…

4. Present your optimum scenario to the class and a representative from the City of Alexandria.

Rubric (TBA)

Additional Performance Tasks and Academic Prompts

(Spiraling in Their Design to Lead Toward Student Understanding and Growing Levels of Transfer): In addition to working on solving equations in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Writing their own equations where the variable equals a certain value (The Calendar Project).
- Students writing their own word problems and solving them.
- Algebraic proofs involving finding the errors in a given equation.
- “I’m thinking of a number."

Other Evidence (quizzes, tests, observations, dialogues, work samples):

1. Tests and Quizzes: Students should complete a minimum of three tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., expression, equation, solution, substitute, evaluate, analyze, solve).

2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to solving equations and inequalities, order of operations, substitution and writing expressions.
Student Self-Assessment and Reflection:

1. Exit Slips: The students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about solving equations and inequalities, order of operations, substitution and writing expressions?

2. Self-Reflection and Self-Assessment Tasks: Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.

3. Accountable Talk: Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (When is less more?) to hook students into considering the effects of math on their lives.
2. Introduce the essential questions and discuss the culminating unit performance tasks (the Potomac Yard “Greenovation”).
3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms. (A possible vocabulary model is the Frayer Model.)
4. Present concept lessons on solving equations and inequalities using different variables and real world situations. Then, have students defend their solutions using the field properties.
5. Working in cooperative groups, students will use various equation solving manipulatives.
6. Students share proposals with members of their group and an expert from the City of Alexandria. Revisions can be made based on the feedback.
7. Students work independently to solve the Calendar Project.
8. Conclude the unit with a quarter one exam.
Suggestions for Scaffolding:

1. Maintain a word wall and notebook log of key vocabulary words along with pictures and student friendly definitions.
2. Use a think-aloud format to demonstrate the steps in solving equations and inequalities.
3. Use algebra tiles as concrete models to represent solving equations. Then assist students in seeing the connection to the symbolic representation.
4. Summarize the process using a substitution check.
5. Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.
6. Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative alternatives for students who can make alternative choices about approach and format.
Stage One: Desired Results

Transfer Goals: The student will...

1. Graph linear equations and linear inequalities in two variables.
2. Design experiments and collect data to address specific, real-world questions.

Enabling Knowledge Objectives (Know/Do):

Declarative Knowledge: Students will know...

1. Key vocabulary terms: slope, x- and y-intercepts, linear equation, rate of change, dependent and independent variable, constant, parallel, perpendicular
2. Changes in slope may be described by rotations or reflections.
3. Changes in the y-intercept may be described by translations, rotations or reflections not across the y-axis.
4. Linear equations can be graphed using slope, x- and y-intercepts, and/or transformations of the parent function.
5. The slope of a line represents a constant rate of change between the independent and dependent variables.
6. The equation of a line defines the relationship between two variables.
7. The graph of a line represents the set of points that satisfies the equation of a line.
8. A line can be represented by its graph or by an equation.
9. The graphing calculator can be used to determine the equation of a line of best fit for a set of data.
10. The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless it is a strict inequality.
11. Parallel lines have equal slopes.
12. The product of the slopes of perpendicular lines is -1 unless one of the lines has an undefined slope.

Essential Questions

1. How can graphs help us to express how things increase or decrease?
2. How can we evaluate the reasonableness of a mathematical model of a real-world situation?

Virginia Standards of Learning

Virginia Standards of Learning (SOLs) in this unit: A.6, A.11 (only linear)
Procedural Knowledge: Students will be able to...

1. Calculate the slope of a line when given an equation of the line, the graph of the line, or two points on the line.
2. Write the equation of a line when given the graph of the line, the slope and a point on the line, or two points on the line whose coordinates are integers.
3. Graph linear equations and inequalities in two variables, including those that arise from a variety of real-world situations.
4. Recognize and describe a line with a slope that is positive, negative, zero, or undefined.
5. Use transformational graphing to investigate effects of changes in equation parameters on the graph of the equation.
6. Write an equation of a vertical line as \( x = a \) and a horizontal line as \( y = c \).
7. Write an equation for a line of best fit, given a set of no more than twenty data points in a table, a graph, or real-world situation.
8. Make predictions about unknown outcomes, using the equation of the line of best fit.

Suggested Resources:

1. Prentice Hall, Algebra I
   - Technology: Functions, Rules and Graphs, p. 253
2. Virginia Department of Education Resources
   - [Algebra I Standards of Learning – 2009 Version](#)
   - [Algebra I Curriculum Framework](#)
   - [Algebra I Enhanced Scope and Sequence](#)
3. Transformationally Thinking
4. Transformation Investigation
5. Slope 2 Slope
6. Inequalities
   - [Mathematics Crosswalk Between the 2009 and 2001 Standards of Learning](#)
   - [Mathematics Vocabulary](#)
   - [Mathalicious](#)
7. NCTM Illuminations
8. Green Globs software
9. Gizmo Math: Go to [www.explorelarning.com](http://www.explorelarning.com) and browse for Gizmo by state correlation. Some examples are:
Defining a Line with Two Points

Distance-Time Graphs
Diagnosis/Pre-Assessment:

Students will complete 8-10 open response problems dealing with calculating slope, identifying y-intercepts, graphing equations of lines, and prediction based on data. The data from this pre-assessment will be used to plan and differentiate for the instruction.

Additionally, diagnosis/pre-assessment should determine students’ readiness levels and background knowledge related to: (1) writing equations given slope, y-intercept or two points, (2) graphing lines given an equation in any form, and (3) finding the line of best fit given a set of data to predict.

Culminating/ Anchor Performance Assessment Task(s)

This project will evaluate student’s ability to collect data, graph and write an equation for the line of best fit, interpret slope and y-intercept in terms of a real world problem and make predictions.

Procedure

1. Investigate salaries of employees based on their minimum education. Go to one of these 3 websites to get your data.

2. Put information on a graph with the x value as the years of school with no High School diploma as 10 years, High School (13), Associates Degree (15), Bachelors (17), and Masters (19). The y value as salary.

3. Find the line of best fit by using two points.

4. Find the line of best fit by using the graphing calculator.
Procedure, continued

5. Discuss the differences between the two different lines.
   - What is the slope of each line?
   - What does the slope mean in the context of this question? (Sample answer: The salary goes up for every year of school).
   - What is the y-intercept of each line?
   - What does the y-intercept mean in the context of this question? (Sample answer: How much money will I make at birth?).
   - Why are the two equations different even though they both use the same data?

6. Predict the salary of certain jobs.
   - Lawyer – threes after Bachelors
   - General Physician – seven years after Bachelors
   - Brain Surgeon – ten years after Bachelors

Additional Performance Tasks and Academic Prompts

(Spiraling in Their Design to Lead Toward Student Understanding and Growing Levels of Transfer): In addition to working on graphing activities in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Real-life word problems involving rate of change. For example, distance equals rate times time, and hourly rates in salary.
- Real-life word problems involving rate of change and y-intercept. For example, bank accounts where you start with a certain amounts of money.
- Textbook problems that make use of varied methodologies, including visual representations and illustrations.
- Academic Prompts: Given a linear scenario define the slope and y-intercept in the context of the problem.
Other Evidence (quizzes, tests, observations, dialogues, work samples):

1. Tests and Quizzes: Students should complete a minimum of three tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., slope, y-intercept, function, transformations, line of best fit)

2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to graphing.

Student Self-Assessment and Reflection:

1. Exit Slips: The students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about graphing equations and inequalities, the meaning of slope and y-intercept?

2. Self-Reflection and Self-Assessment Tasks: Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.

3. Accountable Talk: Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (How much will you have to earn an hour to be rich by age thirty?) to hook students into considering the effects of math on their lives.

2. Introduce the essential questions and discuss the culminating unit performance tasks (The Salary Problem – Line of Best Fit).

3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms. (A possible vocabulary model is the Frayer Model.)

4. Give quizzes on: graphing equations of lines given slope, y-intercept, or two points; interpreting slope and y-intercept; finding the line of best fit and make predictions from data.

5. Students work on Green Globs software independently.

6. Conclude the unit with a student self-evaluation writing assignment on whether or not the data would influence you to get the next higher degree.

Suggestions for Scaffolding:

1. Maintain a word wall and notebook log of key vocabulary words along with pictures and student friendly definitions.

2. Use a think-aloud format to demonstrate the steps in graphing equations and inequalities.

3. Enhance the process using the TI-83 Graphing Calculator.

4. Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.

5. Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative alternatives for students who can make alternative choices about approach and format.
### Algebra 1 - Unit Four  
#### Quarter 2: 4 Weeks

**Putting the "Fun" Back in Functions: Analyzing Linear Functions**

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### Stage One Desired Results

**Transfer Goals: Students will:**

Investigate and analyze linear function families (parent function is \( y = x \)) and their characteristics both algebraically and graphically.

### Enabling Knowledge Objectives (Know/Do):

**Declarative Knowledge: Students will know...**

1. Key vocabulary terms: function, linear function, algebraically, graphically, ordered pairs, domain, range
2. A set of data may be characterized by patterns, and those patterns can be represented in multiple ways.
3. Graphs can be used as visual representations to investigate relationships between quantitative data.
4. Inductive reasoning may be used to make conjectures about characteristics of function families.
5. A relation is a function if and only if each element in the domain is paired with a unique element of the range.
6. The values of \( f(x) \) are the ordinates of the points of the graph of \( f \).
7. The object \( f(x) \) is the unique object in the range of the function \( f \) that is associated with the object \( x \) in the domain of \( f \).
8. For each \( x \) in the domain of \( f \), \( x \) is a member of the input of the function \( f \), \( f(x) \) is a member of the output of \( f \), and the ordered pair \([x, f(x)]\) is a member of \( f \).
9. An object \( x \) in the domain of \( f \) is an \( x \)-intercept or a zero of a function \( f \) if and only if \( f(x) = 0 \).

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### Essential Questions

1. What is a function? How do functions express relationships?
2. How are sets of numbers related to each other?
3. How can we express and analyze linear functions in everyday living situations?

### Virginia Standards of Learning

**Virginia Standards of Learning (SOLs) in this unit: A.7**
**Procedural Knowledge (Do): Students will be able to:**

1. Determine whether a relation, represented by a set of ordered pairs, a table, or a graph is a function.
2. Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically.
3. For each $x$ in the domain of $f$, find $f(x)$.
4. Represent relations and functions using concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, students will be able to represent the relation in another form.
5. Detect patterns in data and represent arithmetic and geometric patterns algebraically.

**Suggested Resources:**

1. Prentice Hall, Algebra I
2. Graphing on the Coordinate Plane, 1.9
3. Relating Graphs to Events, 5.1
4. Writing a Function Rule, 5.4
5. Virginia Department of Education Resources
   - Algebra I Standards of Learning – 2009 Version
   - Algebra I Curriculum Framework
   - Algebra I Enhanced Scope and Sequence
     - Transformationally Thinking
     - Transformation Investigation
   - Mathematics Crosswalk Between the 2009 and 2001 Standards of Learning
   - Mathematics Vocabulary
6. Mathalicious
7. NCTM Illuminations
8. Gizmo Math: Go to [www.explorelearning.com](http://www.explorelearning.com) and browse for Gizmo by state correlation. An example is:

**Introduction to Functions**

![Introduction to Functions Image](image-url)
**Diagnosis/Pre-Assessment:**

Students will complete 8-10 open response problems based on misconceptions dealing with functions. The data from this pre-assessment will be used to plan and differentiate for the instruction. Additionally, diagnosis/pre-assessment should determine students’ readiness levels and background knowledge related to: (1) domain and range, and (2) analyzing functions.

**Culminating/ Anchor Performance Assessment Task(s)**

**The Burning Candle**

It is your friend’s birthday. You want to surprise her by walking into the room carrying a piece of cake with a lighted candle. Can you predict when the candle will go out?

To answer this question, we will use a video presentation (or an actual candle) of a burning candle to collect data, make a graph, and look for a pattern.

1. Gather data from watching a burning birthday candle on a scale and place your data in a table. Note the mass of the candle at various times during the presentation and write down your observations. You should make at least five observations. Be sure to write down both the time and the associated candle mass.

2. Make a new column in your table to show elapsed time, that is, the total time since the candle was lit.

3. On axis (using equal intervals) with elapsed time on the horizontal axis, graph your data, comparing mass, y, to elapsed time, x. Sketch a line or curve by connecting your data points. Compare your graph with those of your team.

4. Use your graph to predict the mass of the candle at the elapsed time of 2 minutes and 47 seconds.
Additional Performance Tasks and Academic Prompts

(Spiraling in Their Design to Lead Toward Student Understanding and Growing Levels of Transfer): In addition to working on function activities in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Real-life word problems involving domain and range of linear functions.
- Models of textbook problems that make use of a range of modalities, including visual representations and illustrations.
- Academic Prompt: Given a real world scenario, establish a reasonable domain and range. Create the equation and graph.

Other Evidence (quizzes, tests, observations, dialogues, work samples):

1. Tests and Quizzes: Students should complete tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., domain, range, function).
2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to functions.

Student Self-Assessment and Reflection:

1. Exit Slips: Students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about analyzing functions? Define domain and range give a real world scenario.
2. Self-Reflection and Self-Assessment Tasks: Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.
3. Accountable Talk: Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (How can our analysis of linear functions such as the burning candle activity support efficiency in everyday situations?) to hook students into considering the effects of math on their lives.

2. Introduce the essential questions and discuss the culminating unit performance tasks (The Burning Candle Problem).

3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms. (A possible vocabulary model is the Frayer Model.)

4. Present concept lessons on domain and range in real world situations. Then, have students compare the results for their representation.

5. Working in cooperative groups, students create 5-10 graphs in a given family (same type of slope). Groups exchange graphs and write the corresponding equations. Teacher observes and coaches students as they work.

6. Each student designs an illustrated function brochure to teach other algebra students about families of linear functions. This activity is completed outside of the class.

7. Students share brochures with members of their group for a peer assessment based on criteria. Allow students to make revisions based on feedback.

8. Conclude the unit with a student self-evaluation regarding their understanding of function families using real life scenarios as examples.
1. Maintain a word wall and notebook log of key vocabulary words along with pictures and student friendly definitions.

2. Use TI-83 Graphing Calculator as a concrete model to visualize families of functions. Then assist students in seeing the connection to the symbolic representation.

3. Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.

4. Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative alternatives for students who can make alternative choices about approach and format.
Algebra 1 - Unit Five  
Quarter 2: 3-4 Weeks  
Please be Direct! Analyzing Direct and Inverse Variation

Stage One Desired Results

Transfer Goals: The student will:

1. Analyze a relation to determine whether a direct or inverse variation exists.
2. Represent a direct variation algebraically and graphically and an inverse variation algebraically.

Enabling Knowledge Objectives (Know/Do):

Declarative Knowledge: Students will know...

1. Key vocabulary terms: variation, direct variation, inverse variation, dependent variable, independent variable,
2. The constant of proportionality in a direct variation is represented by the ratio of the dependent variable to the independent variable.
3. The constant of proportionality in an inverse variation is represented by the product of the dependent variable and the independent variable.
4. A direct variation can be represented by a line passing through the origin.
5. Real-world problems may be modeled using direct and/or inverse variations.

Procedural Knowledge: Students will be able to...

1. Given a situation, including a real-world situation, determine whether a direct variation exists.
2. Given a situation, including a real-world situation, determine whether an inverse variation exists.
3. Write an equation for a direct variation, given a set of data.
4. Write an equation for an inverse variation, given a set of data.
5. Graph an equation representing a direct variation, given a set of data.

Essential Questions

1. How can we create real-life models to express variant situations?
2. How can we express situations that do not vary directly?

Virginia Standards of Learning (SOLs)

Virginia Standards of Learning (SOLs) in this unit: A.8

Direct and Inverse Variation
Suggested Resources:

1. Prentice Hall, Algebra I
   - Inverse Variation: 12.1
2. Virginia Department of Education Resources
   - Algebra I Standards of Learning – 2009 Version
   - Algebra I Curriculum Framework
   - Algebra I Enhanced Scope and Sequence
     - Direct Variation
     - Inverse Functions
   - Mathematics Crosswalk Between the 2009 and 2001 Standards of Learning
   - Mathematics Vocabulary
3. Gizmo Math: Inverse Variation Activities
4. Mathalicious
5. NCTM Illuminations: Do I Have to Mow the Whole Thing?
Stage Two: Assessment Evidence

**Diagnosis/Pre-Assessment:**

Students will complete 3-6 open response problems based on misconceptions dealing with analyzing a relationship to determine if it is a direct or inverse variation. The data from this pre-assessment will be used to plan and differentiate for the instruction. Additionally, diagnosis/pre-assessment should determine students’ readiness levels and background knowledge related to: (1) analyzing relationships, (2) determining if it is direct or inverse, and (3) the representing the relationship algebraically.

**Culminating/ Anchor Performance Assessment Task(s)**

**Estimating Fish Populations**

Fish biologists need to keep track of fish populations in the waters they monitor. They want to know, for example, how many striped bass there are in San Francisco Bay. This number changes throughout the year as fish move in and out of the bay to spawn. Therefore biologists need a way to gather current data fairly quickly and inexpensively. Since it is impossible to count every animal biologists use the “tag and recapture” process to help them estimate the size of a population.

Tag and recapture involves collecting a sample of animals, tagging them, and releasing them back into the wild. Later, biologists collect a new sample of the animals and count the number in the sample distinguishing between first time captures and recaptures. They then use ratios to estimate the population size.

Each team has a lake (paper sack) full of fish (beans), and a net (a small cup). Be sure each team member records all the team’s data, calculations, and conclusions.
OVERVIEW

Your task is to estimate the number of fish in the lake using the technique used by the Department of Fish and Game. Here is a preview of this simulation.
Each team has a lake (paper sack) full of fish (beans), and a net (a small cup). Be sure each team member records all the team’s data, calculations, and conclusions.
   1. Collect an initial sample. Count and tag the specimens.
   2. Return the tagged sample to the lake.
   3. Collect an additional sample. Count the number of tagged and untagged fish.
   4. Use the data (counts) to write proportions.
   5. Solve the ratios to find the lake population.

When you finish these steps you will have three pieces of data to help you calculate the total number of fish in the lake:
   1. total number of tagged fish in the lake
   2. number of tagged fish in the sample and
   3. total number of fish in the sample

Now you have the data needed to find the total number of fish in the lake. Follow the directions below to guide you through the process.
Write an equation that incorporates these four data pieces into a proportion on the resource page.

<table>
<thead>
<tr>
<th>Total number of tagged fish in the lake</th>
<th>Number of tagged fish in sample</th>
<th>Total number of fish in the sample</th>
<th>Total number of fish in the lake</th>
</tr>
</thead>
</table>

1. Use your “net” to collect a sample (cupful) of fish (beans) from the lake. Count the number of fish you netted in your sample.
2. To “tag” the netted fish, either mark them with a marking pen or substitute a bean of a different color for each fish as directed by your teacher. This is the only sample you will tag in this simulation.
3. On the Resource Page record this number as the “total number of tagged fish in the lake.”
4. Put all the tagged fish back into the lake. Be careful not to let any of the fish jump out on to the floor. Gently shake the bag to thoroughly mix all the fish in the lake. Try not to bruise them!
COLLECTING A SAMPLE

1. Use your net to take out another cupful of fish. Count and record the “total number of fish in sample.”

2. Count and record the number of “number of tagged fish in sample.”

ESTIMATING THE POPULATION

1. You now have three pieces of information: the total number of tagged fish in the lake, the total number of fish in the sample, and the number of tagged fish in the sample.

2. Use this information to write a proportion and solve for the total number of fish in the lake.

3. Return your sample of fish to the lake. Gently mix the fish. Take another sample. Repeat the counting and recording procedures of parts (c) and (d). Use the new information to write a proportion as in part (e). Solve the equation you wrote for the total number of fish in the lake.

Extension:

Your solutions represent two estimates for the population of your lake. While it is important to get an accurate count of the fish population, each time you net a sample it costs the taxpayers $500 for your time and equipment. So far your samples have cost $1000. If you feel your estimate is accurate at this point, record it on the class chart with our cost. If you think you should try another sample for better accuracy, do the same steps as before. Draw as many samples as need, but remember each sample costs $500.

1. Record your team’s data on the class chart. Then count the fish in your lake to find the actual population. Record the actual population on the class chart.

2. Was your estimate close? If not, what might have thrown it off? What do you think of this method for estimating fish population?
Additional Performance Tasks and Academic Prompts

(Spiraling in Their Design to Lead Toward Student Understanding and Growing Levels of Transfer): Prior to working on determining direct and inverse variation activities in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Real-life word problems involving student created scenarios of graphs with direct and inverse variation.
- Real-life data displayed in tables that students can discuss and rationalize if the situation is similar to direct or indirect variation.
- Models of real data displayed in scatterplots that students can discuss and rationalize how the situation is similar and different from graph models of direct and inverse variation.
- Real data displayed as word problems so that students can discuss and rationalize how the situation is similar and different from graph models of direct and inverse variation.
- Models of textbook problems that make use of a range of modalities, including visual representations and illustrations.

Other Evidence (quizzes, tests, observations, dialogues, work samples):

1. Tests and Quizzes: Students should complete a minimum of three tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., variation, constant of variation, direct variation, inverse variation, asymptote, independent variable, dependent variable, proportion)

2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to direct and inverse variation.
Student Self-Assessment and Reflection:

1. Exit Slips: The students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about why data is categorized as direct or inverse variation?

2. Self-Reflection and Self-Assessment Tasks: Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.

3. Accountable Talk: Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (How do we categorize data?) to hook students into considering the effects of math on their lives.

2. Introduce the essential questions and discuss the culminating unit performance tasks (Estimating Fish Populations).

3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms. (A possible vocabulary model is the Frayer Model.)

4. Introduce examples and non-examples of the rationale for why dependent and independent variables are standardized on the x/y axis.

5. Working in cooperative groups, students will collect, investigate, and graph problematic situations. Then categorize them into direct variation, inverse variation, and other to develop a definition for direct and inverse variation.

6. Students create their own real-life scenarios where they predict an outcome of direct variation, inverse variation, or neither. Then, they test their hypothesis and summarize their learning.
7. Use VA Enhanced Scope and Sequence fraction lesson, “Direct Variation”, to engage students in real life scenarios.

8. Each student designs an illustrated “data in our world” brochure to teach adults how to use direct and inverse variation to predict patterns and rationalize real-life situations. This activity is completed outside of the class.

9. Students share brochures with members of their group for a peer assessment based on criteria. Allow students to make revisions based on feedback.

10. Students listen to, and question, a mathematician about what why we look for patterns and categorize graphs.

11. Conclude the unit with a quarter two exam.
Suggestions for Scaffolding:

1. Maintain a word wall and notebook log of key vocabulary words along with pictures and kid friendly definitions.

2. Use a think-aloud format to demonstrate prediction, analysis, and conclusion of determining if a set of data can be categorized and direct or inverse variation.

3. Use graphs and data from current events, news, and other media.

4. Use a graphic organizer to make connections between

5. X-axis, dependent variable, and domain

6. Y-axis, independent variable, and range

7. Constant of variation, direct variation, visual of the graph, table, and formula

8. Inverse variation, visual of the graph, table, and formula

9. Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.

10. Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative alternatives for students who can make alternative choices about approach and format.
Stage One: Desired Results

**Transfer Goals: The student will:**

1. Solve systems of two linear equations in two variables algebraically and graphically.
2. Solve systems of two linear inequalities in two variables.

**Enabling Knowledge Objectives (Know/Do):**

**Declarative Knowledge: Students will know...**

1. Key vocabulary terms: system of linear equations, inequality, infinite, unique solution
2. A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations.
3. A system of two linear equations with no solution is characterized by the graphs of two lines that are parallel.
4. A system of two linear equations having infinite solutions is characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and the coordinates of all points on the line satisfy both equations.
5. Systems of two linear equations can be used to model two real-world conditions that must be satisfied simultaneously.
6. Equations and systems of equations can be used as mathematical models for real-world situations.

**Essential Questions**

1. What is a system? How can equations form systems?
2. How can a system of equations and inequalities support you in solving real-life world problems?

**Virginia Standards of Learning**

*Virginia Standards of Learning (SOLs) in this unit: A.4, A.5*

**Solving Systems of Linear Equations**
Procedural Knowledge: Students will be able to...

1. Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to find the ordered pair which satisfies both equations.
2. Given a system of two linear equations in two variables that has a unique solution, solve the system graphically by identifying the point of intersection.
3. Determine whether a system of two linear equations has one solution, no solution, or infinite solutions.
4. Write a system of two linear equations that models a real-world situation.
5. Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a real-world situation.

Suggested Resources:

1. Zaccaro
   - Chapter 17
2. Prentice Hall
   - Solving Systems by Graphing, 7.1
   - Solving Systems Using Substitution, 7.2
   - Solving Systems Using Elimination, 7.3
3. Virginia Department of Education Resources
   - Algebra I Standards of Learning – 2009 Version
   - Algebra I Curriculum Framework
   - Algebra I Enhanced Scope and Sequence
   - Road Trip
   - The Exercise Ring
   - Mathematics Crosswalk Between the 2009 and 2001 Standards of Learning
   - Mathematics Vocabulary
4. Mathalicious
5. NCTM Illuminations
6. Gizmo Math: Go to [www.explorelearning.com](http://www.explorelearning.com) and browse for Gizmo by state correlation. An example is: Modeling Linear Systems
Diagnosis/Pre-Assessment:

Students will complete 3-6 open response problems based on misconceptions dealing with solving systems. The data from this pre-assessment will be used to plan and differentiate for the instruction. Additionally, diagnosis/pre-assessment should determine students’ readiness levels and background knowledge related to: (1) graphing linear functions, (2) using substitution or elimination, and (3) using systems to solve real world questions.

Culminating/Anchor Performance Assessment Task(s)

Choosing a Phone Plan

You have decided to try and convince your parents to let you get your own telephone line. After finding out that the monthly rate is $13.50, which you can afford, you offer to pay the cost yourself. However, just before you place your order, you see an advertisement that the local telephone companies are discontinuing the flat rate of $13.50 per month and will only offer metered service. Their ads are shown below:

- **Flat Rate Cancelled!**
  - $13.50 per month is longer available

- **AT²**
  - 2 cents per minute + $3 each minute

- **PaBell's Plan!**
  - 1 cent per minute + $8.50 each month
Your parents think that you need to do a little research before making a decision. What would you do to help make your decision?

- About how many minutes of phone calls do you make per month? What is the largest number of minutes from your team?

- For the flat rate and for each company make a table comparing total minutes to total cost. Choose at least four reasonable amounts to represent the number of minutes per month. It will be helpful to include 0 minutes and the largest number of minutes from your team. You will have three tables when you are done.

- We are now going to plot our data. Carefully plan how to scale your axes. Label the vertical axis “cost per month” and the horizontal axis “minutes of calls” per month. Plot and connect the points for each table in part (b). Be sure to label the graph of each company plan.

- When do both companies charge the same amount? When is AT$^2$ a better choice? When is a Pa Bell a better choice? Discuss how your graphs related to the solution of the problem.

- Consider the options using the number of minutes you call per month. Which plan would you choose? Will you save money over the flat rate?

Additional Performance Tasks and Academic Prompts

(Spiraling in Their Design to Lead Toward Student Understanding and Growing Levels of Transfer): Prior to working on systems of equations activities in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Real-life word problems involving systems of equation and the prediction of where the equations intersect.

- Models of textbook problems that make use of a range of modalities, including visual representations and illustrations.

- Students writing their own word problem system of equations and solving them.
Other Evidence (quizzes, tests, observations, dialogues, work samples):

1. Tests and Quizzes: Students should complete a minimum of three tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., system of linear equations, inequality, infinite, unique solution).

2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to systems of equations.

Student Self-Assessment and Reflection:

1. Exit Slips: At the conclusion of at least three classes per week, students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about solving system of equations?

2. Self-Reflection and Self-Assessment Tasks: Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.

3. Accountable Talk: Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Stage Three: Learning Plan

Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (Is it always the better deal?) to hook students into considering the effects of math on their lives.

2. Introduce the essential questions and discuss the culminating unit performance tasks (Choosing a Phone Plan).

3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms using the Frayer Model.

4. Present concept lessons on systems of equations and real world situations. Then, have students defend scenarios of when a function represents a “better deal”.

5. Working in cooperative groups, students will design a function for a real-life scenario, eg: cell phone plan. They will compare their function with other group members and display multiple functions on a graph. Then using knowledge of systems of equations, determine which scenario is the “better deal” for various domains.

6. Use VA Enhanced Scope and Sequence fraction lesson, “Road Trip,” “What’s your call?”, “Spring Fling Carnival,” and “The exercise ring,” to engage students in real life systems of equations.

7. Students listen to, and question, a mathematician about when important decisions are made based on systems of equations.

8. Students respond to written prompt: When two scenarios are given, such as in a given system of equations, what strategies do you use to determine which function to choose? Given a real-life situation like the cell phone plan, how do you justify your decision to choose a plan?

9. Conclude the unit with a student self-evaluation regarding their understanding of finding the solution to a system of equations, and their reasoning for choosing one function over the other in a real-life scenario.
Suggestions for Scaffolding:

1. Maintain a word wall and notebook log of key vocabulary words along with pictures and kid friendly definitions.

2. Use a think-aloud format to demonstrate the steps in choosing one cell phone plan, realizing they are equal, then choosing the other plan.

3. Use three color highlighters to identify areas of the data (tables, graphs and word problems) when one function is the better deal, the solution that makes both functions equal, and when the other function is the better deal.

4. Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.

5. Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative alternatives for students who can make alternative choices about approach and format.
Transfer Goals: Students will:

Investigate and solve linear programming problems.

Enabling Knowledge Objectives (Know/Do):

**Declarative Knowledge: Students will know...**

1. Key vocabulary terms: linear programming, minimum, maximum, optimization, constraints, objective quantity
2. Many real-life problems involve a process called optimization, which means finding the minimum and maximum value of some quantity. One type of optimization is called linear programming.
3. A linear programming problem consists of a system of linear inequalities called constraints and an objective quantity, such as a cost or a profit that can be minimized or maximized
4. Constraints define the region of possible answers.
5. The objective quantity is evaluated at the vertices formed by the constraints.

**Procedural Knowledge: Students will be able to...**

1. Solve linear programming problems.
2. Graph linear programming problems.
3. Use linear programming to model real-life problems.

Essential Questions

1. How can we use linear programming to model real-life situations?

Virginia Standards of Learning

*Virginia Standards of Learning (SOLs) in this unit: A.4e*
Suggested Resources:

2. NCTM Illuminations: Dirt Bike Dilemma
3. Linear Programming Power Point
4. Virginia Department of Education Resources
   - Algebra I Standards of Learning – 2009 Version
   - Algebra I Curriculum Framework
   - Algebra II Enhanced Scope and Sequence
     - What is Linear Programming?
     - Linear Programming
     - Linear Programming Problem: Maximizing Tent Production
     - Traveling Trains and Non-Linear Systems
   - Mathematics Crosswalk Between the 2009 and 2001 Standards of Learning
   - Mathematics Vocabulary
5. Gizmo Math: Linear Programming Activities
6. Mathalicious
Algebra 1 - Unit Seven  
Quarter 3: 3-4 Weeks

I Have My Ups and Downs: Using Linear Programming

Stage Two: Assessment Evidence

Diagnosis/Pre-Assessment:

Students will complete 3-6 open response problems based on misconceptions dealing with linear programming. The data from this pre-assessment will be used to plan and differentiate for the instruction. Additionally, diagnosis/pre-assessment should determine students’ readiness levels and background knowledge related to: (1) graphing functions, (2) systems of equations, (3) graphing linear inequalities and (4) reading a graph for meaning.

Culminating/Anchor Performance Assessment Task(s)*

(Designed to evaluate the extent to which students have achieved transfer/understanding at an independent level relative to key mastery objectives and major unit understandings/essential questions):

Video Game Designer

As a video game designer, it is your job to program a video game that includes several characters to choose from, and many levels of game play. You will use linear programming models determine the amount of time you should spend on creating characters, and creating new levels in the game. There are several deadlines that require you to have completed x number of characters, and y number of levels.

In one day, you can make either 4 levels, or 2 characters. Use the Gizmo Linear Programming Activity A to set the objective. Your manager has given you some constraints to meet your deadline, but has also given you options on how you use the rest of your time.

- Constraint 1: In week 1, you must spend 2 days creating characters.
- Constraint 2: In week 1, you must spend 2 days creating levels.
- Constraint 3: In week 1, you may work no more than 6 days total.

What is the maximum number of design items (characters or levels) that you can design in 6 days? Your manager gives you a deadline to create 15 items (characters or levels) by the end of day 5 to have a full weekend. Is this possible? How is it possible?
**Game Design:**

In cooperative groups, decide the complexity of your game. What is the minimum and maximum number of characters and levels you would like the programmer to create? Give constraints for weeks 1 – 4+. Map out a suggested pace for the programmer.

**Additional Performance Tasks and Academic Prompts**

(Spiraling in Their Design to Lead Toward Student Understanding and Growing Levels of Transfer): Prior to working on linear programming activities in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Real-life word problems involving multiple situations, the prediction of what situation is optimal, and the exploration of the linear programming Gizmo tool.

- Models of textbook problems that make use of a range of modalities, including visual representations and illustrations.

- Writing their own situations with multiple constraints, to determine possible combinations and optimal combination.

**Other Evidence (quizzes, tests, observations, dialogues, work samples):**

1. Tests and Quizzes: Students should complete a minimum of three tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., linear programming, minimum, maximum, optimization, constraints, objective quantity)

2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to linear programming.
Student Self-Assessment and Reflection:

1. Exit Slips: At the conclusion of at least three classes per week, students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about linear programming?

2. Self-Reflection and Self-Assessment Tasks: Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.

3. Accountable Talk: Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (How do we decide how to spend our time when we need to do two different types of things?) to hook students into considering the effects of math on their lives.

2. Introduce the essential questions and discuss the culminating unit performance tasks (Video Game Designer).

3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms using the Frayer Model.

4. Working in cooperative groups, students create situations where linear programming is needed to determine efficiency of different combinations.

5. Have groups share their linear programming situations, and discuss as a class.

6. Conclude the unit with a student self-evaluation regarding their understanding of linear programming using real life scenarios as examples.
Suggestions for Scaffolding:

1. Maintain a word wall and notebook log of key vocabulary words along with pictures and kid friendly definitions.

2. Use a think-aloud format to demonstrate the meaning of the solution of a function $f(x,y)$.

3. Use tiles or pattern blocks as concrete models to represent a given combination and total solution to the function. Then assist students in seeing the connection to the symbolic representation.

4. Summarize combinations, minimums, maximums, and optimal quantities using a table.

5. Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.

6. Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative alternatives for students who can make alternative choices about approach and format.
Stage One: Desired Results

**Transfer Goals: The student will:**

Investigate and solve exponential functions problems.

### Enabling Knowledge Objectives (Know/Do):

### Declarative Knowledge (Know): Students will know:

1. Key vocabulary terms: square root, cube root, radicand, perfect square, factors, consecutive integers, exponent, exponential growth, exponential function, growth and decay factor
2. A square root in simplest form is one in which the radicand (argument) has no perfect square factors other than one.
3. A cube root in simplest form is one in which the argument has no perfect cube factors other than one.
4. The cube root of a perfect cube is an integer.
5. The cube root of a non-perfect cube lies between two consecutive integers.
6. The inverse of cubing a number is determining the cube root.
7. In the real number system, the argument of a square root must be nonnegative while the argument of a cube root may be any real number.
8. Exponential functions and graphs display varying rates of change.
9. An exponential function is a multiplication function. As the x-values in the domain increase by a constant amount, the y-values are multiplied by a common multiplier.
10. When the independent variable of a function appears as an exponent of the growth factor, the function is called an exponential function.

### Essential Questions

1. When do quantities have a nonlinear relationship? What’s the significance if they do?
2. How do we compare the difference between linear and exponential growth?
3. How can exponential functions be used to model real-life situations?

### Virginia Standards of Learning

*Virginia Standards of Learning (SOLs) in this unit: A.2a, A.3*
Procedural Knowledge: Students will be able to...

1. Express square roots of a whole number in simplest form.
2. Express the cube root of a whole number in simplest form.
3. Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values.
4. Simplify monomial expressions and ratios of monomial expressions in which the exponents are integers.
5. Solve an exponential function for a specified variable.
6. Confirm algebraic solutions of an exponential function, using a graphing calculator.
7. Graph exponential functions from numerical data.
8. Determine the growth and decay factor for an exponential function represented by a table of values or an equation.

Suggested Resources:

1. Zaccaro
   - Chapter 8: Exponents, Radicals and Scientific Notation
2. Prentice Hall
   - Exponents and Exponential Functions, 8.1-8.8
3. Virginia Department of Education Resources
   - Algebra I Standards of Learning – 2009 Version
   - Algebra I Curriculum Framework
   - Algebra I Enhanced Scope and Sequence
     - Exponents
   - Mathematics Crosswalk Between the 2009 and 2001 Standards of Learning
   - Mathematics Vocabulary
4. Gizmo Math: Exponential Function Activities
5. Mathalicious
6. NCTM Illuminations
Stage Two: Assessment Evidence

Diagnosis/Pre-Assessment:

Students will complete 3-6 open response problems based on misconceptions dealing with exponential expressions. The data from this pre-assessment will be used to plan and differentiate for the instruction. Additionally, diagnosis/pre-assessment should determine students’ readiness levels and background knowledge related to: (1) graphing functions, and (2) reading a graph for meaning.

Culminating/ Anchor Performance Assessment Task(s)

Who Wants to be Rich?

Unit Eight Academic Prompts

This assignment will evaluate student’s declarative knowledge of investigating and solving exponential functions. It will allow students to compare and contrast linear and exponential functions. These models will show how these functions follow real world applications.

Linear Academic Prompt Procedure

Your task is to create a quiz show called Who wants to be Rich? Contestants will be asked a series of questions. A contestant will play until he or she misses a question. The total prize money will grow with each question answered correctly.

- Lucy proposes that a contestant receive $5 for answering the first question correctly. For each additional correct answer, the total prize would increase by $10
- How many questions would a contestant need to answer correctly to win at least $50? To win at least $75? To win at least $100?
- Sketch a graph of the (n, p) data for n = 1 to 10.
Exponential Academic Prompt

Pedro also thinks the first question should be worth $5. However, he thinks a contestant’s winnings should double with each subsequent correct answer.

1. For Pedro’s proposal, what equation gives the total prize \( p \) for correctly answering \( n \) questions?
2. How many questions will a contestant need to answer correctly to win at least $50? To win at least $75? To win at least $100?
3. Sketch a graph of the \((n, p)\) data for \( n = 1 \) to 10
4. Compare Pedro’s proposal with Lucy’s proposal in the linear prompt

Additional Performance Tasks and Academic Prompts

(Spiraling in Their Design to Lead Toward Student Understanding and Growing Levels of Transfer): Prior to working on exponential functions in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Real-life word problems involving exponents in growth and decay.
- Models of textbook problems that make use of a range of modalities, including visual representations and illustrations.
- Students writing their own problems using exponential functions, then solving them.

Other Evidence (quizzes, tests, observations, dialogues, work samples):

1. Tests and Quizzes: Students should complete a minimum of three tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., square root, cube root, radicand, perfect square, factors, consecutive integers, exponent, exponential growth, exponential function, growth and decay factor)
2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to exponential functions.
### Student Self-Assessment and Reflection:

1. **Exit Slips:** At the conclusion of at least three classes per week, students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about how linear functions differ from exponential functions?

2. **Self-Reflection and Self-Assessment Tasks:** Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.

3. **Accountable Talk:** Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (How do employees get raises? Is it always linear?) to hook students into considering the effects of math on their lives.

2. Introduce the essential questions and discuss the culminating unit performance tasks (Who Wants to Win?)

3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms using the Frayer Model.

4. Present concept lessons on exponential functions using different real world situations. Then, have students relate their experiences to this model.

5. Students create their own scenarios of exponential growth or decay, then make predictions, and solve using tables, graphs, and rules.

6. Use VA Enhanced Scope and Sequence fraction lesson, “Exponents”, to engage students in real life pattern scenarios. Included at the end of the lesson are several articles of when exponential functions are used in the world.

7. Read and discuss the book, “The King’s Chessboard.” The book tells a story of how a servant is paid 1 grain of rice on day 1, 2 grains on day 2, 4 grains on day 3, 8 grains on day 4, 16 grains on day 5, etc. Students can predict outcomes along the way.

8. Conclude the unit with a student self-evaluation regarding their understanding of finding how exponential functions differ from linear functions, and where they are found in the world.

Suggestions for Scaffolding

1. Maintain a word wall and notebook log of key vocabulary words along with pictures and kid friendly definitions.

2. Use a think-aloud format to demonstrate the exponential change and how is changes dramatically.
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<table>
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<td>3.</td>
<td>Use poster sized graph paper so that students can visualize how the graph maintains its shape.</td>
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<td>4.</td>
<td>Use computer models to zoom in and out of a graph when finding points.</td>
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<td>5.</td>
<td>Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.</td>
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<tr>
<td>6.</td>
<td>Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative alternatives for students who can make alternative choices about approach and format.</td>
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Algebra 1 - Unit Nine
Data is Everywhere! Analyzing and Graphing Data

Stage One: Desired Results

Transfer Goals:
Analyze descriptive statistics to determine the implications for the real-world situations from which the data derive.
Use statistical techniques to organize, display, and compare sets of data.
Determine variance and standard deviation of a data set and interpret the standard deviation.
Investigate data and create graphs.

Enabling Knowledge Objectives (Know/Do):

Declarative Knowledge: Students will know...

1. Key vocabulary terms: statistics, measures of center, variance, standard deviation, absolute variation, mean, dispersion, z-score, box-and-whisker plot
2. Descriptive statistics may include measures of center and dispersion.
3. Variance, standard deviation, and mean absolute deviation measure the dispersion of the data.
4. The sum of the deviations of data points from the mean of a data set is 0.
5. Standard deviation is expressed in the original units of measurement of the data.
6. Standard deviation addresses the dispersion of data about the mean.
7. Standard deviation is calculated by taking the square root of the variance.
8. The greater the value of the standard deviation, the further the data tends to be dispersed from the mean.
9. For a data distribution with outliers, the mean absolute deviation may be a better measure of dispersion than the standard deviation or variance.

Essential Questions
1. How does current consumer data affect how we make purchases?
2. How do graphs assist us in making decisions in our everyday lives?
3. How do we decide on which type of graph to use to present data?
4. How does standard deviation address dispersion?

Virginia Standards of Learning (SOLs)
Virginia Standards of Learning (SOLs) in this unit: A.9, A.10

Hurricane Motion
10. A z-score derived from a particular data value tells how many standard deviations that data value is above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean.

11. A z-score (standard score) is a measure of position derived from the mean and standard deviation of data.

12. Box-and-whisker plots can be used to analyze data.

**Procedural Knowledge: Students will be able to...**

1. Given data, including data in a real-world context, calculate and interpret the mean absolute deviation of a data set.
2. Given data, including data in a real-world context, calculate variance and standard deviation of a data set and interpret the standard deviation.
3. Given data, including data in a real-world context, calculate and interpret z-scores for a data set.
4. Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.
5. Compare and contrast mean absolute deviation and standard deviation in a real-world context.
6. Compare, contrast, and analyze data, including data from real-world situations displayed in box-and-whisker plots.
7. Given data, including data in a real-world context, calculate and interpret the mean absolute deviation of a data set.
8. Given data, including data in a real-world context, calculate variance and standard deviation of a data set and interpret the standard deviation.
9. Given data, including data in a real-world context, calculate and interpret z-scores for a data set.
10. Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.
11. Compare and contrast mean absolute deviation and standard deviation in a real-world context.
12. Compare, contrast, and analyze data, including data from real-world situations displayed in box-and-whisker plots.
Suggested Resources:

1. Mean and Standard Deviation Investigations
2. Virginia Department of Education Resources
4. Algebra I Curriculum Framework
5. Algebra I Enhanced Scope and Sequence
6. Mathematics Crosswalk Between the 2009 and 2001 Standards of Learning
   • Mathematics Vocabulary
7. Gizmo Math: Box-and-Whisker Plots
8. Mathalicious
9. NCTM Illuminations
Diagnosis/Pre-Assessment:

Students will complete 3-6 open response problems based on misconceptions dealing with representing and analyzing patterns. The data from this pre-assessment will be used to plan and differentiate for the instruction. Additionally, diagnosis/pre-assessment should determine students’ readiness levels and background knowledge related to: (1) measures of central tendancy, (2) representing statistical data on a graph, and (3) Using statistical data to summarize.

Culminating/ Anchor Performance Assessment Task(s)

* (Designed to evaluate the extent to which students have achieved transfer/understanding at an independent level relative to key mastery objectives and major unit understandings/essential questions):

Jump Rope Battle

You are in charge of a jump rope competition. All the men and women in your class will compete, but inevitable, there will likely be the disagreement as to who jumps rope “better”, the men or the women. You will need to collect data. Your job is to investigate this idea of “better.” To do this, you will find the mean, median, mode, range, minimum, maximum, standard mean, standard deviation, and z-scores. You will use this data to crown one gender the “better”.

1. Create a scatterplot for each data set on the following graphs.
   - Draw in the mean line (y=mean) with a pen or colored pencil.
   - Draw vertical lines from the points to the mean. Why do we need to think about this distance?
   - Using this distance determine a square for each and then estimate the area of the average square.
2. How does this value compare to the variance and/or the standard deviation?
3. Determine intervals and counts – fill in the table on the first page.
   - Create a histogram below each scatter plot.
4. Determine the standard deviation for each data set as a sample of all men (women).
5. Interpret in writing the standard deviation for each data set.
6. Determine which gender is indeed the “better” Jump Roper.
Additional Performance Tasks and Academic Prompts

(Spiraling in Their Design to Lead Toward Student Understanding and Growing Levels of Transfer): Prior to working on statistics activities in a text, students will participate in a series of performance tasks guided by the instructor. Such tasks will include:

- Real-life word problems involving statistics, predictions, and conclusions based on the mathematics.
- Models of textbook problems that make use of a range of modalities, including visual representations and illustrations.

Other Evidence (quizzes, tests, observations, dialogues, work samples):

1. Tests and Quizzes: Students should complete a minimum of three tests and/or quizzes focusing on key unit elements, including key academic vocabulary (e.g., statistics, measures of center, variance, standard deviation, absolute variation, mean, dispersion, z-score, box-and-whisker plot).

2. Games: Teachers monitor students as they engage in both hands-on and electronic versions of games related to statistics.

Student Self-Assessment and Reflection:

1. Exit Slips: At the conclusion of at least three classes per week, students will complete a brief closure activity involving their reactions to a teacher-posed question: e.g., What do you understand better now about how to analyze statistics?

2. Self-Reflection and Self-Assessment Tasks: Periodically, students will be asked to reflect on how he or she has contributed to his or her own learning process. For example, students may be given a checklist and are asked to evaluate if they need to work harder or need instructional support on a particular topic or focus area.

3. Accountable Talk: Throughout the unit, students should be involved in tasks that require metacognition and self-regulation, ensuring that they monitor their own progress relative to unit understandings and mastery objectives. The goal here is to have the learner actively involved in self-assessment and responsible self-talk, rather than having the teacher always determine the quality of his or her performance.
Consider the sequence of key teaching-assessment-learning tasks and instructional elements.

1. Begin with an entry question (How do we use statistics in our everyday lives?) to hook students into considering the effects of math on their lives.

2. Introduce the essential questions and discuss the culminating unit performance tasks (Jump Rope Battle).

3. Note: Key vocabulary terms are introduced as needed by the various learning activities and performance tasks. Students read and discuss word problems and highlight the terms to support the learning activities and tasks. As an ongoing activity, students keep a notebook of important terms using the Frayer Model.

4. Present concept lessons on representing statistics in real world situations. Then, have students compare the results for their representations.

5. Investigate current events, news, and other media to identify statistics, and re-evaluate the data to determine if the message has been manipulated.

6. Students listen to, and question, a mathematician about what how statistics are needed in our world, but how they can be manipulated into different meanings.

7. Conclude the unit with a final exam.

Suggestions for Scaffolding:

1. Maintain a word wall and notebook log of key vocabulary words along with pictures and kid friendly definitions.

2. Summarize vocabulary terms, formulas, and examples using a graphic organizer.

3. Be sure that there are levels of challenge that will meet the needs of all students in reaching their maximum potential.
4. Explore ways that the culminating cornerstone/anchor performance assessment task can be designed to allow independent and creative alternatives for students who can make alternative choices about approach and format.